Paralelos Mundo Contínuo x Mundo Digital

Respostas de sistemas (plano-s × plano-z)

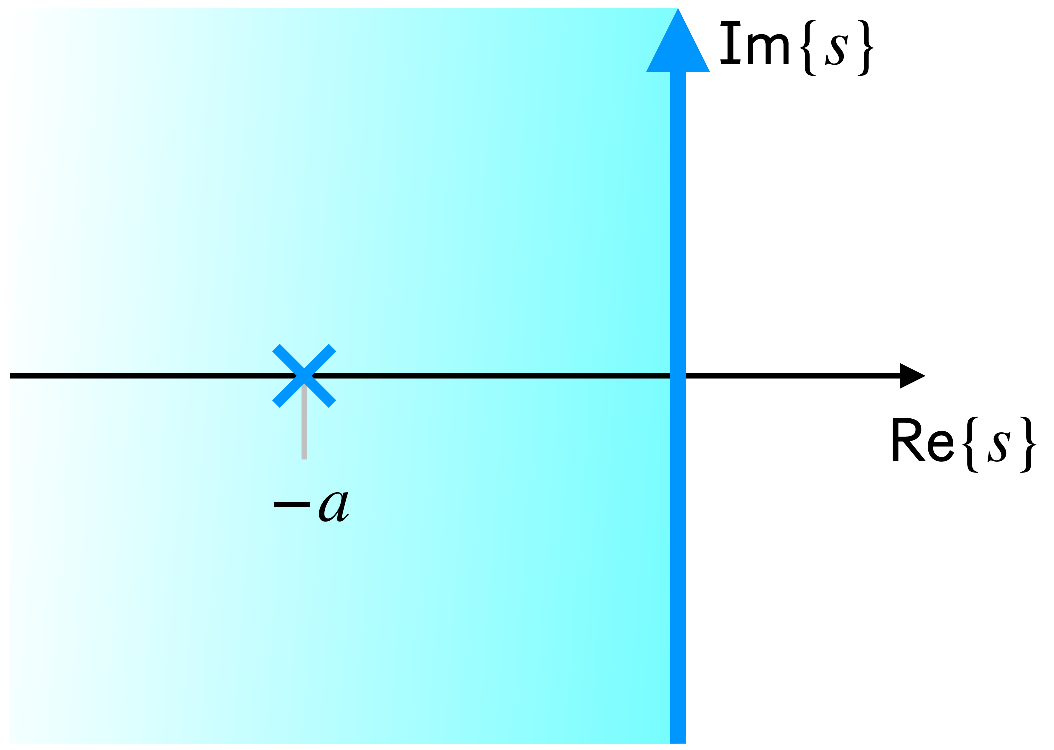
Prof. Fernando Passold

Respostas de Sistemas de 1a-ordem

No plano-s ("mundo contínuo"):

Sistema 1 pólo real simples, exemplo:

$$G(s) = \frac{A}{s+a}$$



Obs.: $y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s)$.

Obs.: $\mathscr{L}^{-1}\left\{ \frac{a}{s(s+a)} \right\} = (1 - e^{-at})$.

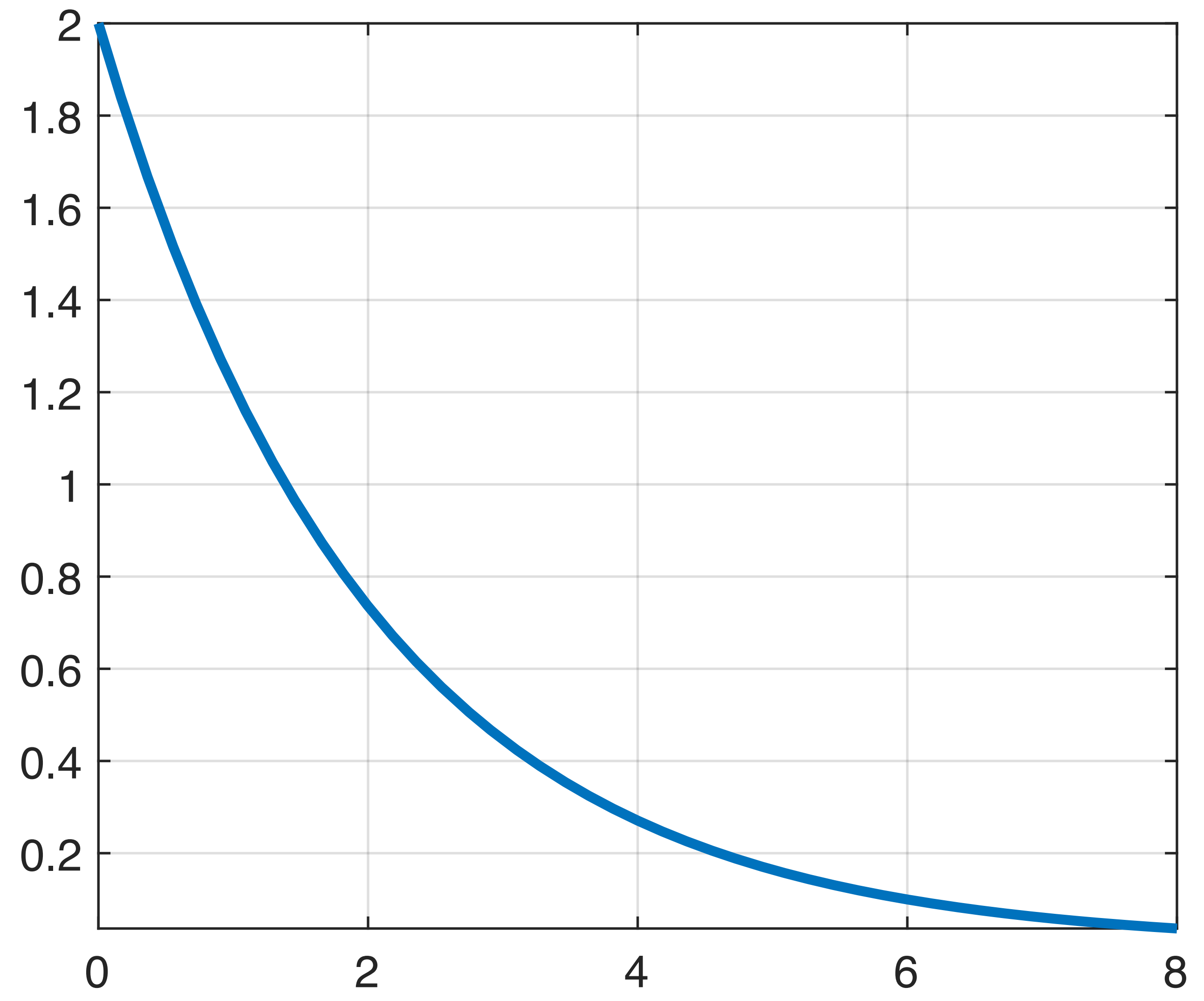
→Resposta ao Impulso:

$$Y(s) = \delta(s) \cdot G(s)$$

$$y(t) = \mathscr{L}^{-1}\{\delta(s) \cdot G(s)\} = 1 \cdot A \cdot e^{-at}$$

Ex.: $Y(s) = \dfrac{2}{s + 1/2}$

```
>> fplot(@(t) 2*exp(-0.5*t), [0 8])
```



→Resposta ao Degrau:

$$Y(s) = U(s) \cdot G(s)$$

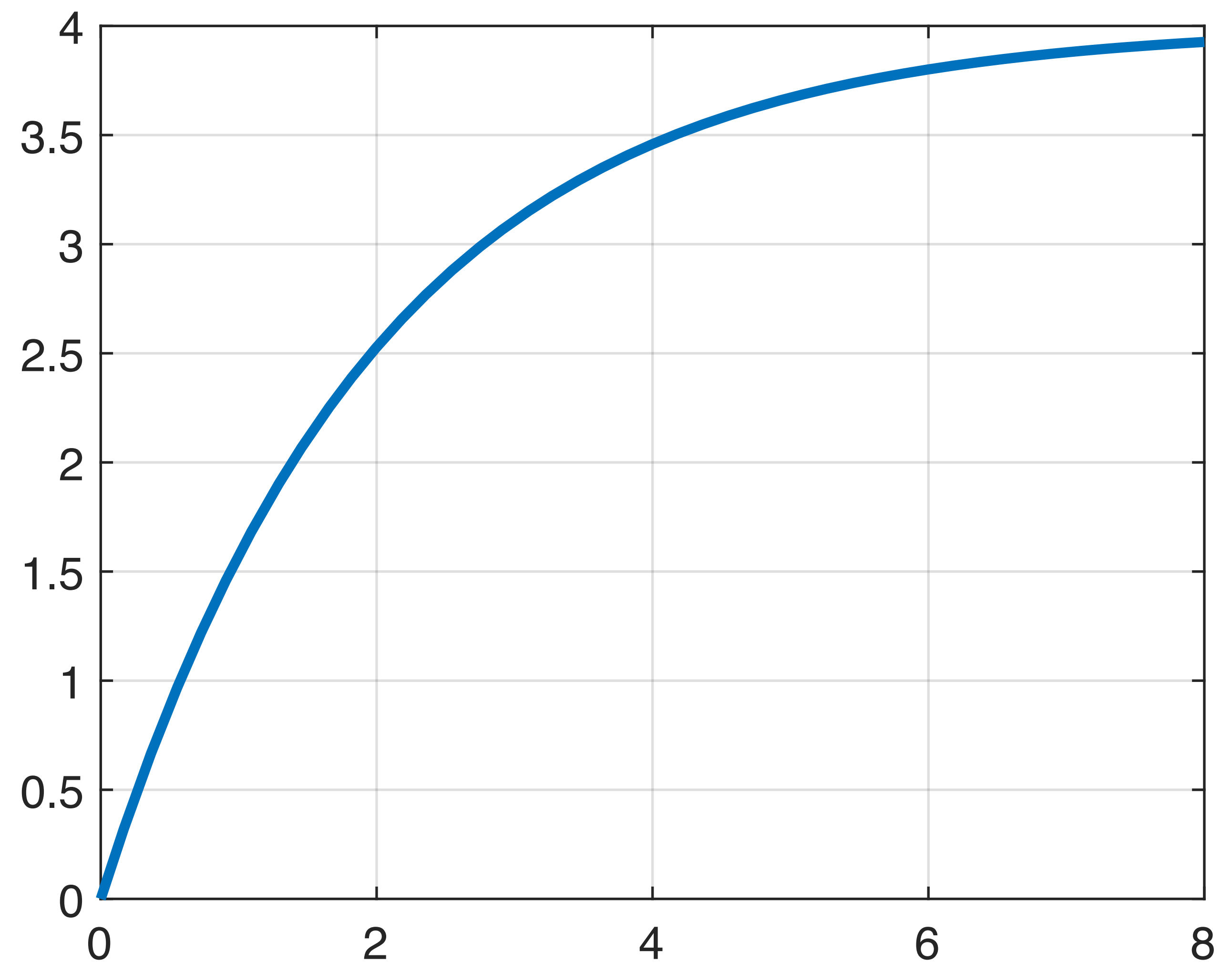
$$Y(s) = \left(\frac{1}{s}\right)\left(\frac{A}{s+a}\right) = \frac{A}{s(s+a)}$$

$$y(t) = \mathscr{L}^{-1}\{U(s) \cdot G(s)\}$$

$$y(t) = \frac{A}{a}\left(1 - e^{-at}\right)$$

Ex.: $Y(s) = \dfrac{1}{s} \cdot \dfrac{2}{s + 1/2}$

```
>> fplot(@(t) (2/0.5)*(1-exp(-0.5*t)), [0 8])
```

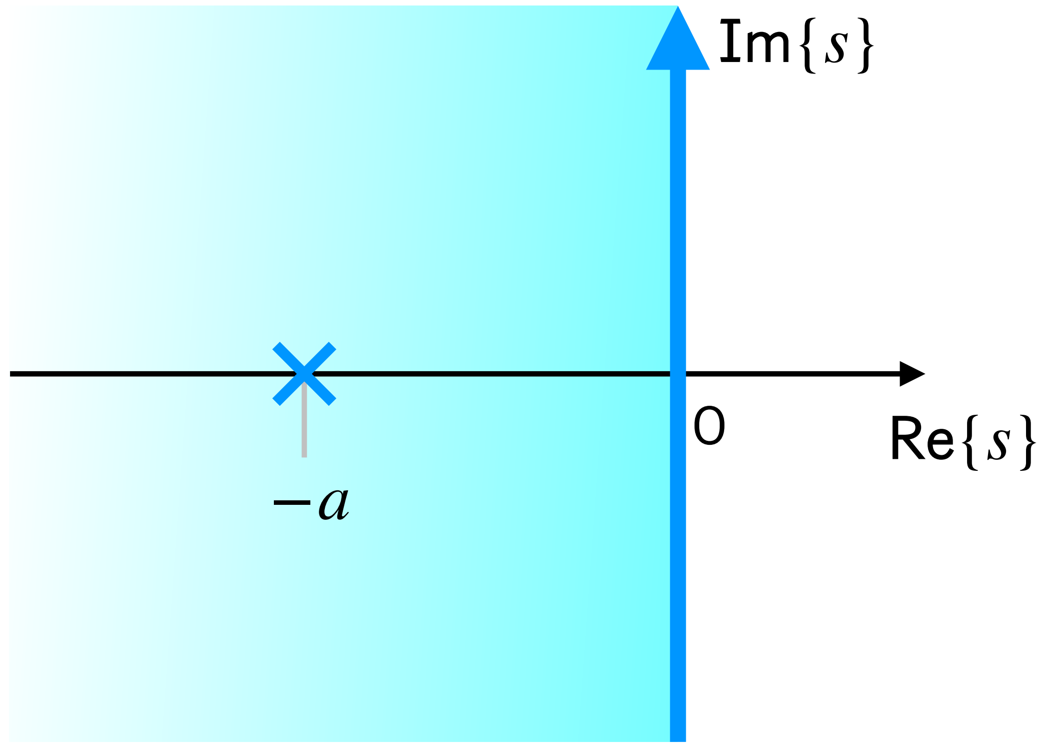


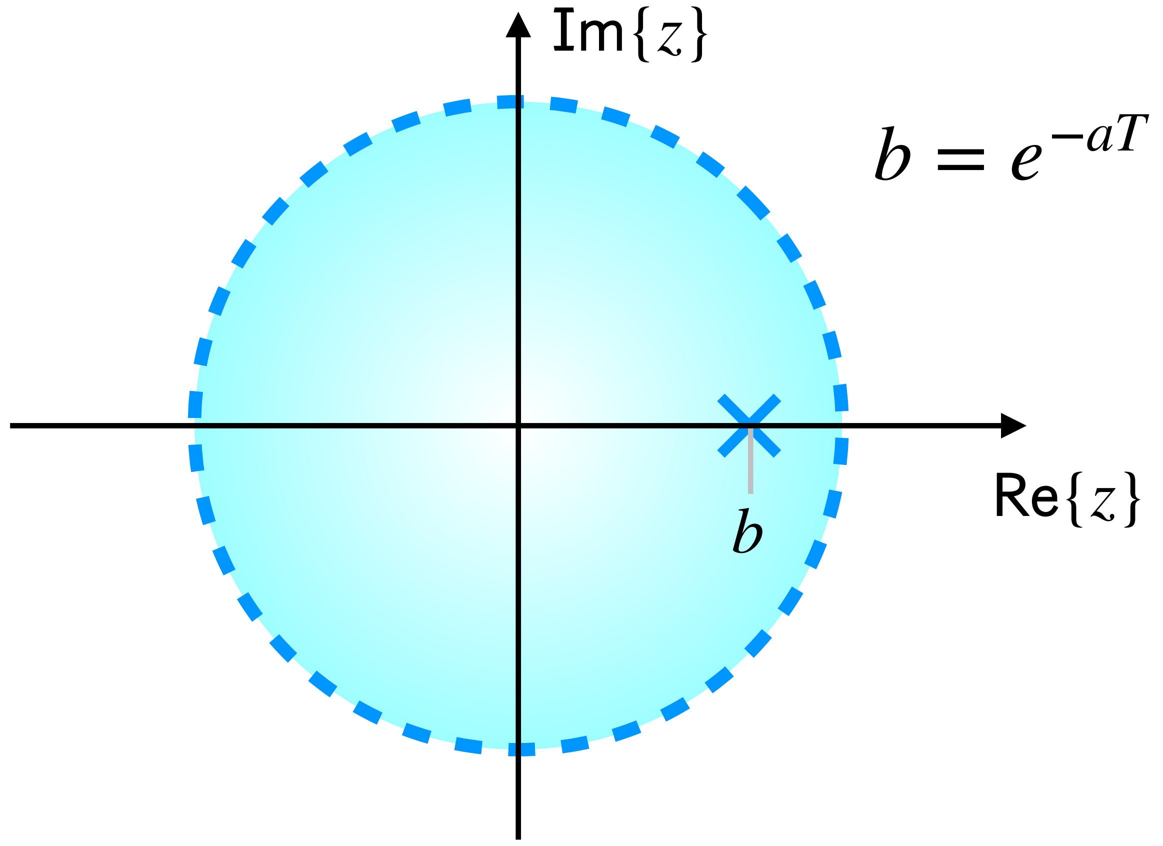
Respostas de Sistemas de 1a-ordem

No plano-z (“mundo discreto”):

Sistema 1 pólo real simples, exemplo:

$$G(s) = \frac{A}{s+a}$$





$$b = e^{-aT}$$

Ex.: $Y(s) = \dfrac{2}{s+1/2}$

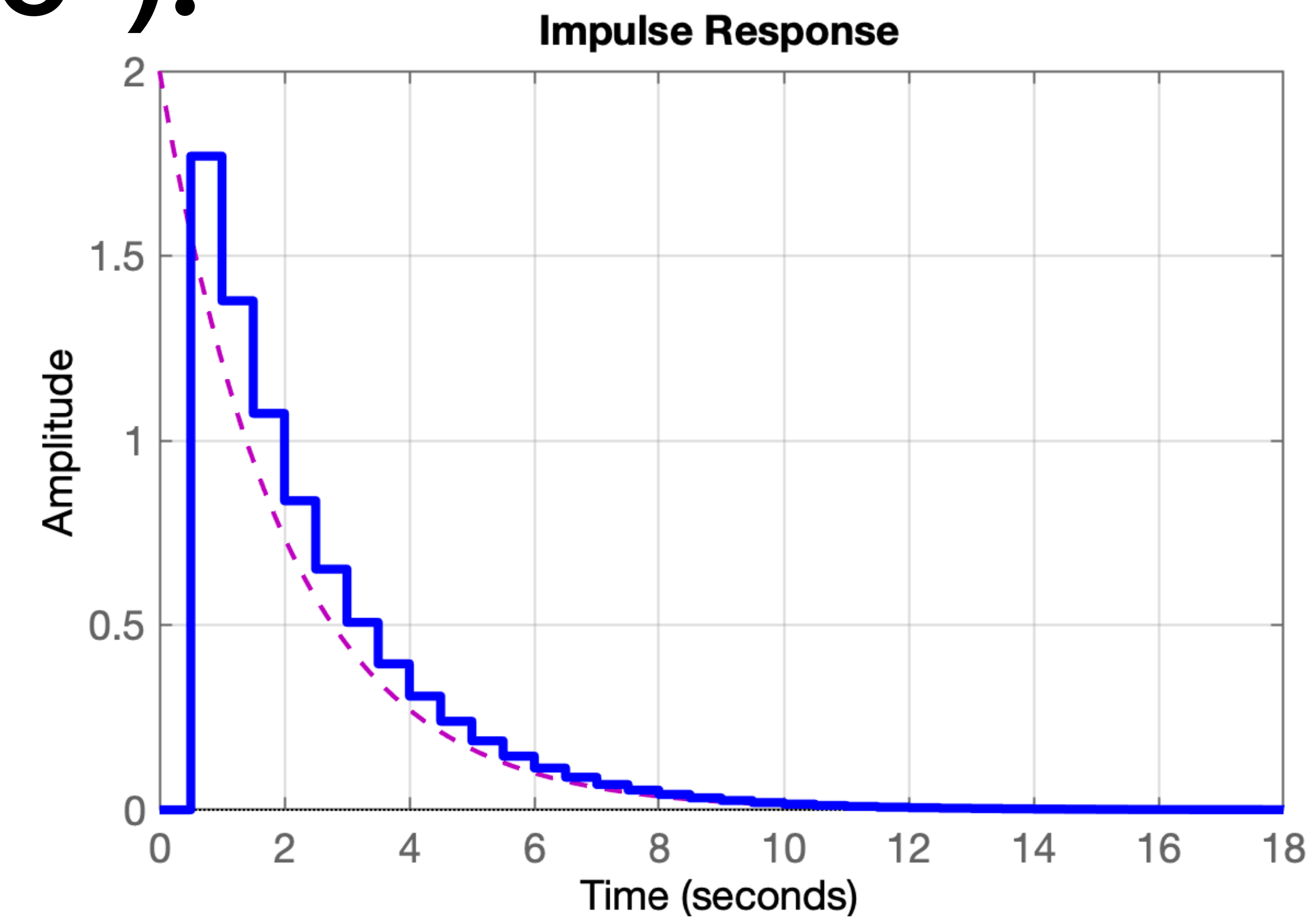
T=0,5 segundos:

$$BoG(z) = \frac{0{,}8848}{(z-0{,}7788)}$$

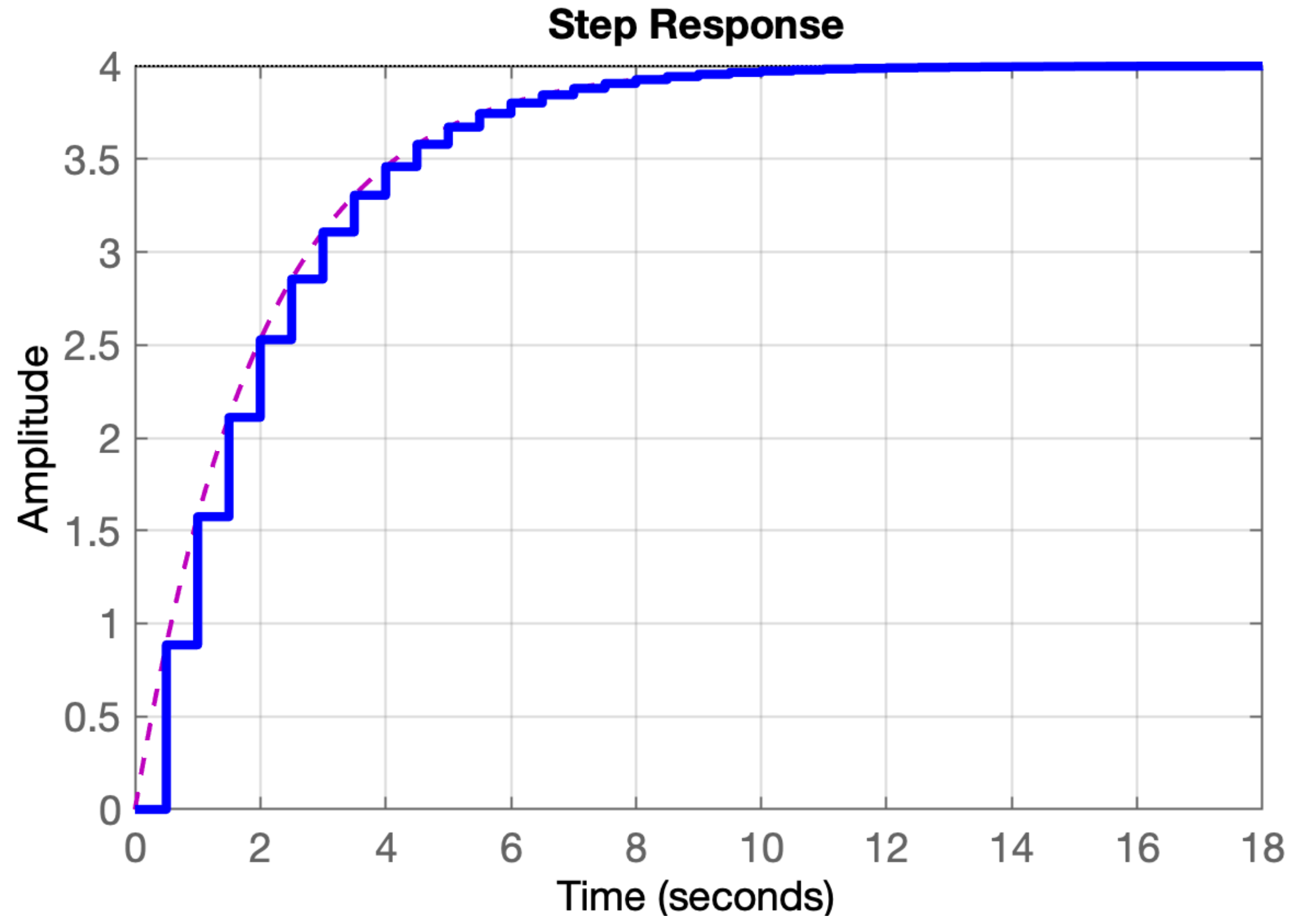
Matlab:

```
>> G=tf(2,[1 1/2]);
>> zpk(G)
     2
  -------
  (s+0.5)
Continuous-time zero/pole/gain model.
>> T=0.5;
>> BoG=c2d(G,T);
>> zpk(BoG)
    0.8848
  ----------
  (z-0.7788)
Sample time: 0.5 seconds
Discrete-time zero/pole/gain model.
>> figure; impulse(G,'m--', BoG,'b-')
>> figure; step(G,'m--', BoG,'b-')
>> a=1/2;
>> b=exp(-a*T)
b =       0.7788
>>
```

→Resposta ao Impulso:



→Resposta ao Degrau:



Respostas de Sistemas de 1a-ordem

No plano-s ("mundo contínuo"):

Sistema 1 pólo real simples, exemplo:

$$G(s) = \frac{A}{s+a}$$

→Resposta ao Impulso:

$$y(t) = \mathscr{L}^{-1}\{\delta(s) \cdot G(s)\} = 1 \cdot A \cdot e^{-at}$$

→Resposta ao Degrau:

$$y(t) = \frac{A}{a}\left(1 - e^{-at}\right)$$

Obs.: $y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot Y(s)$.

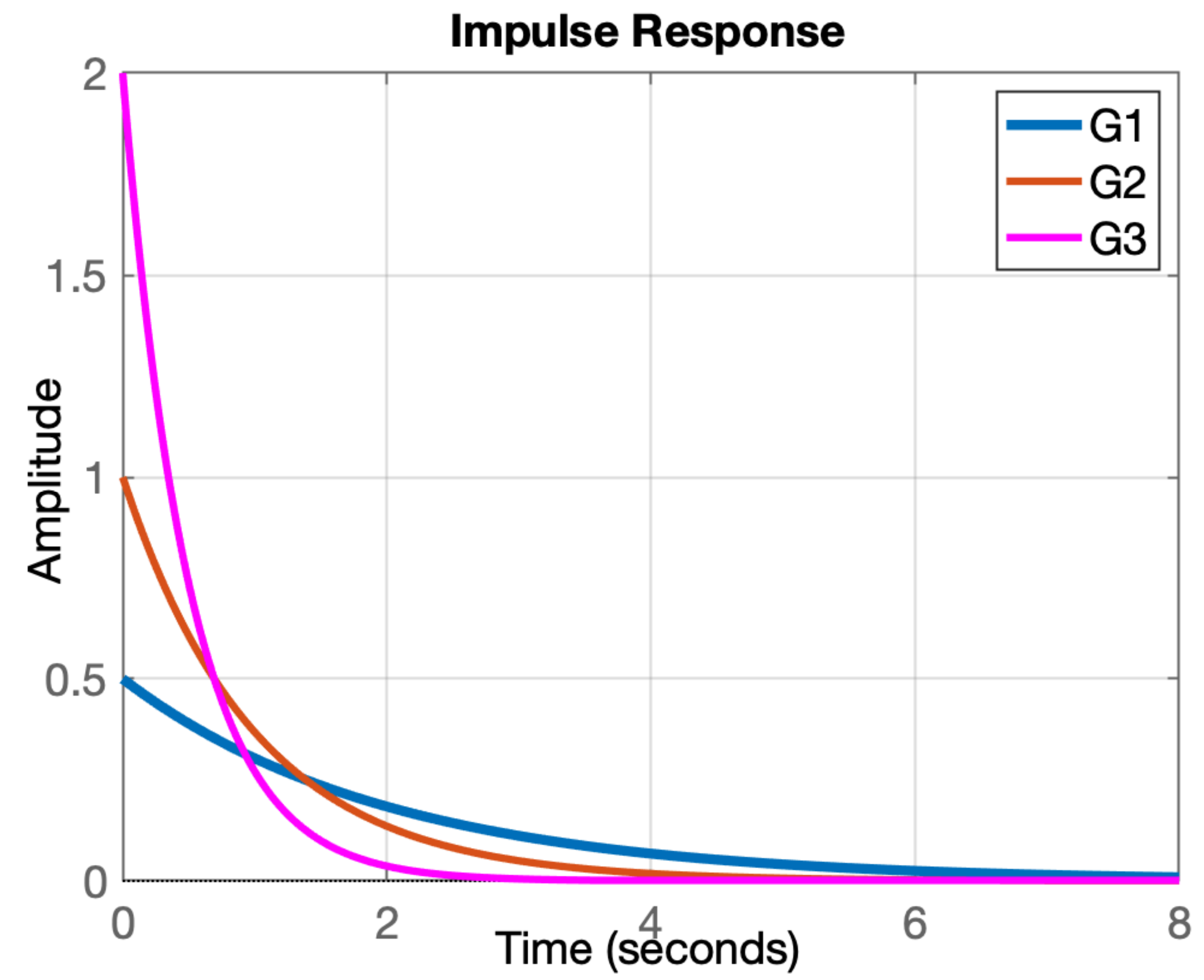
Obs.: $\mathscr{L}^{-1}\left\{\frac{a}{s(s+a)}\right\} = (1 - e^{-at})$.

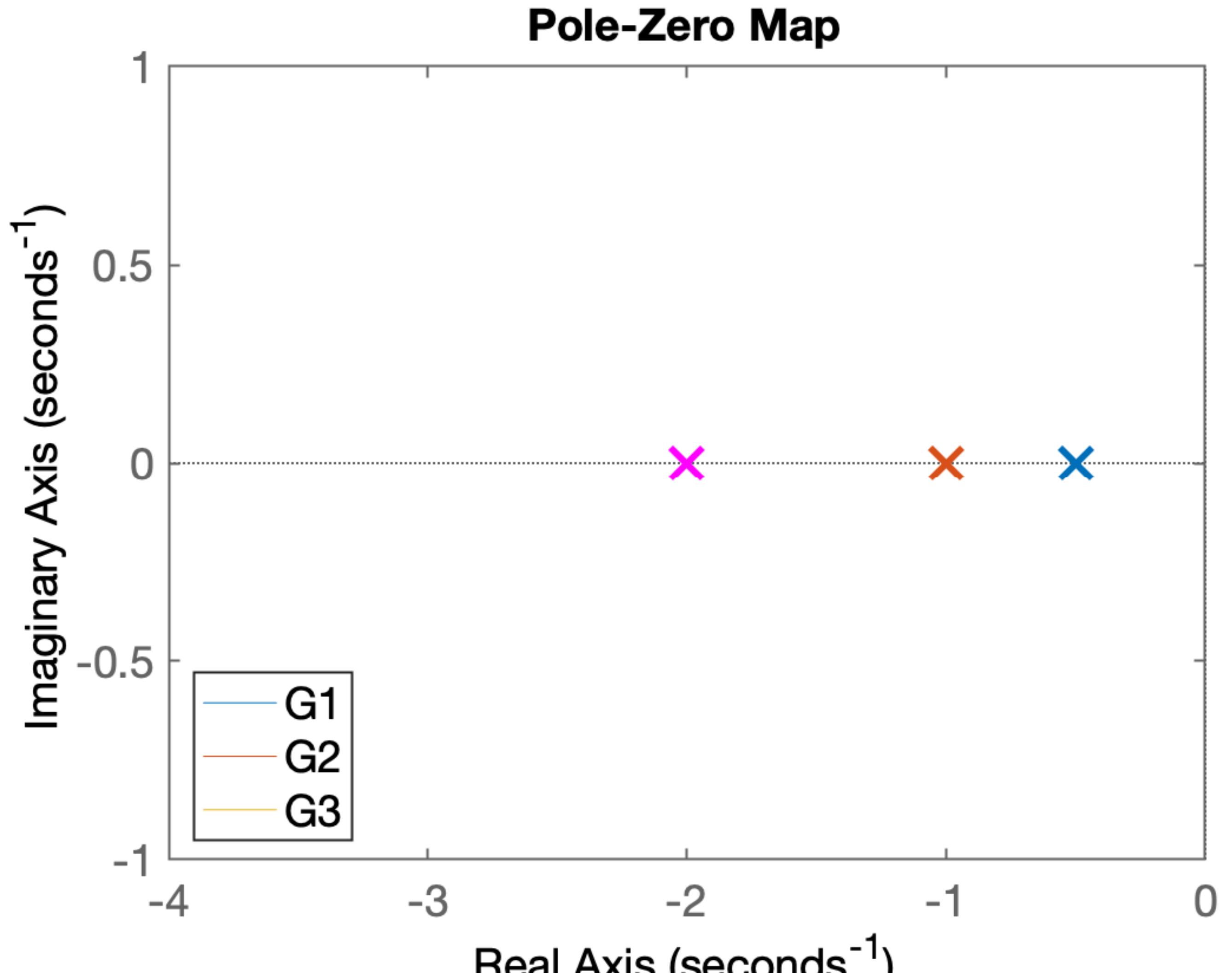
Exemplo:

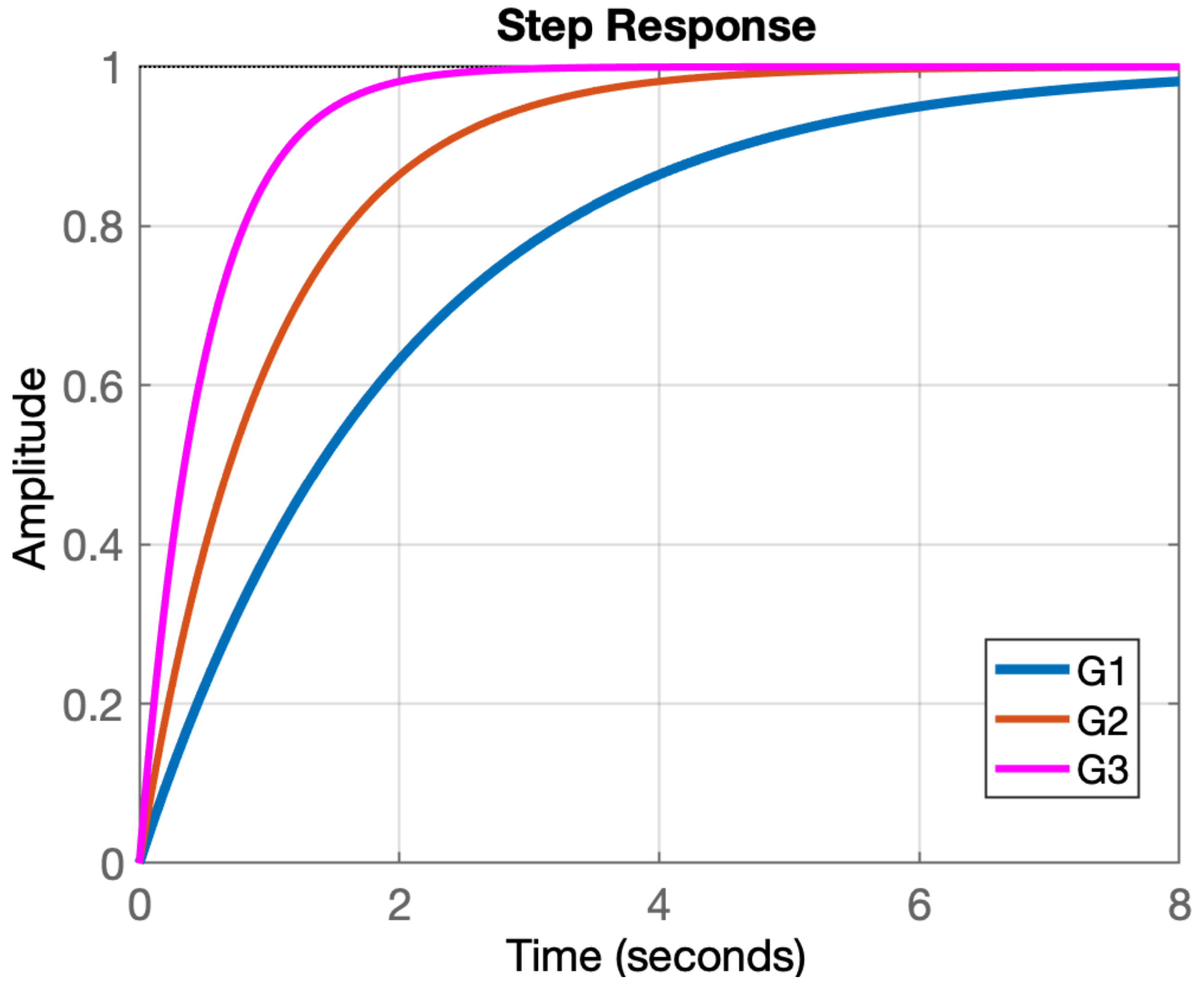
$$G_1(s) = \frac{1/2}{s+1/2}$$

$$G_2(s) = \frac{1}{s+1}$$

$$G_3(s) = \frac{2}{s+2}$$







Respostas de Sistemas de 1a-ordem

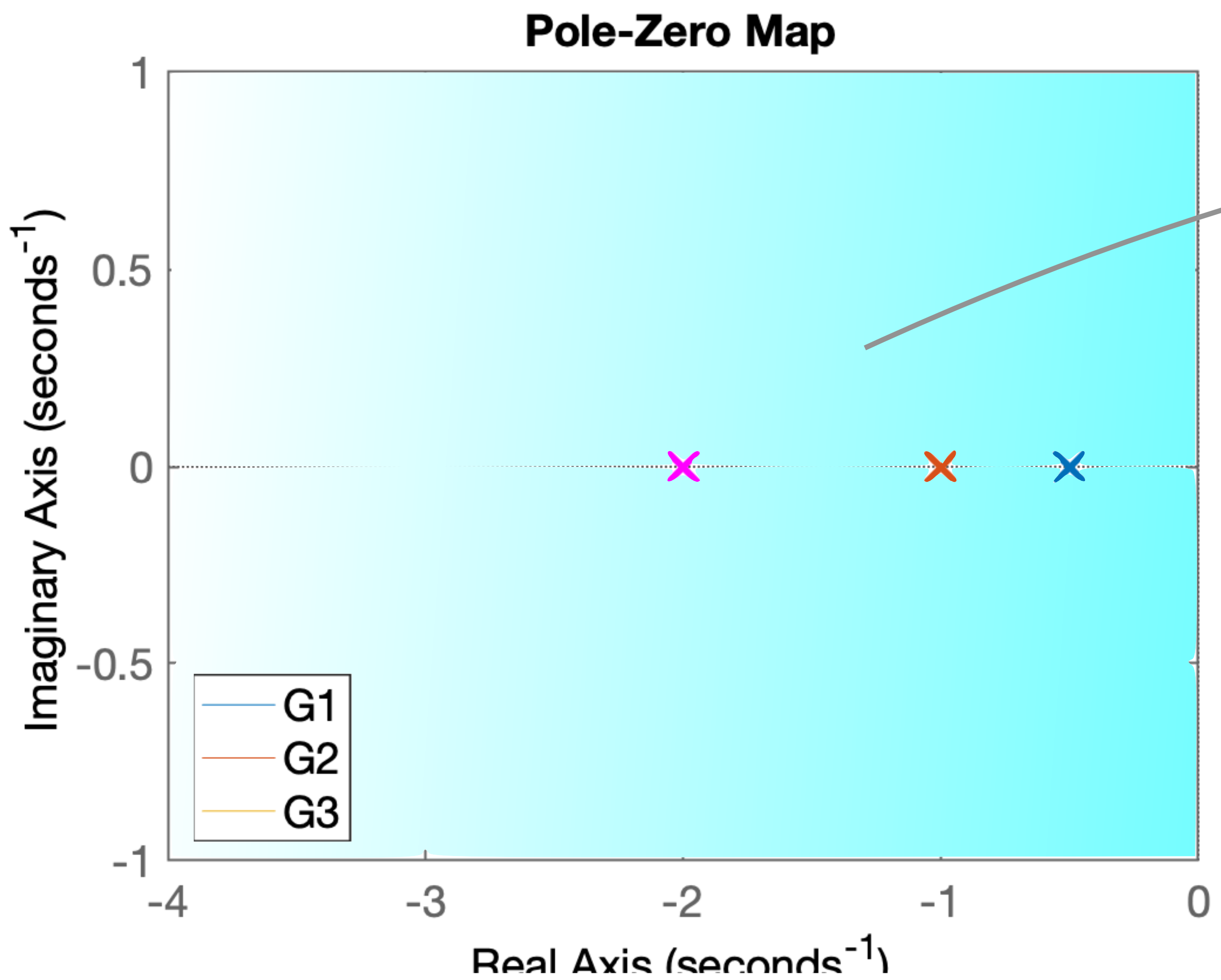
No plano-z ("mundo discreto"):

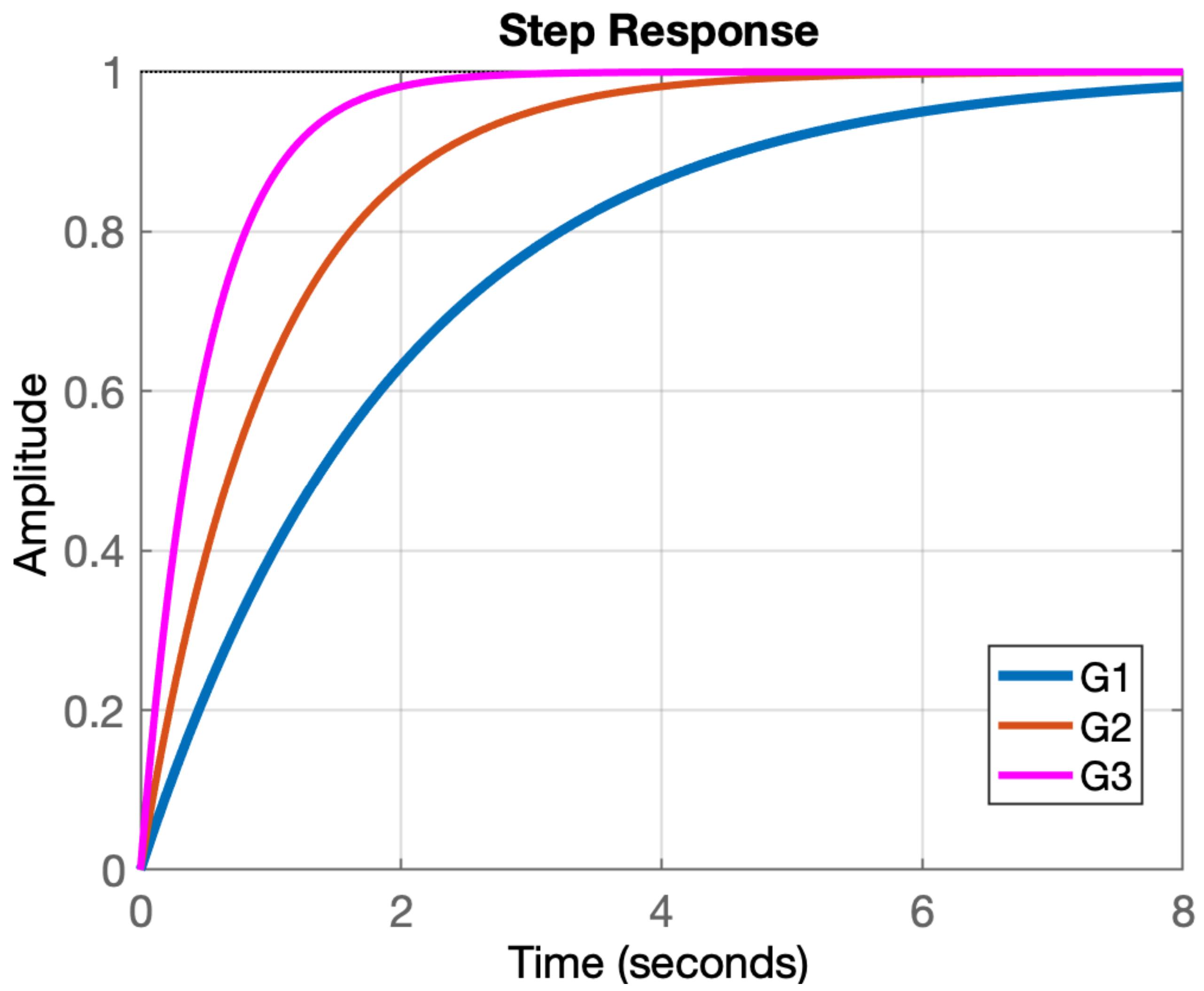
Exemplo:

$$G_1(s) = \frac{1/2}{s + 1/2}$$

$$G_2(s) = \frac{1}{s + 1}$$

$$G_3(s) = \frac{2}{s + 2}$$





T=0,5

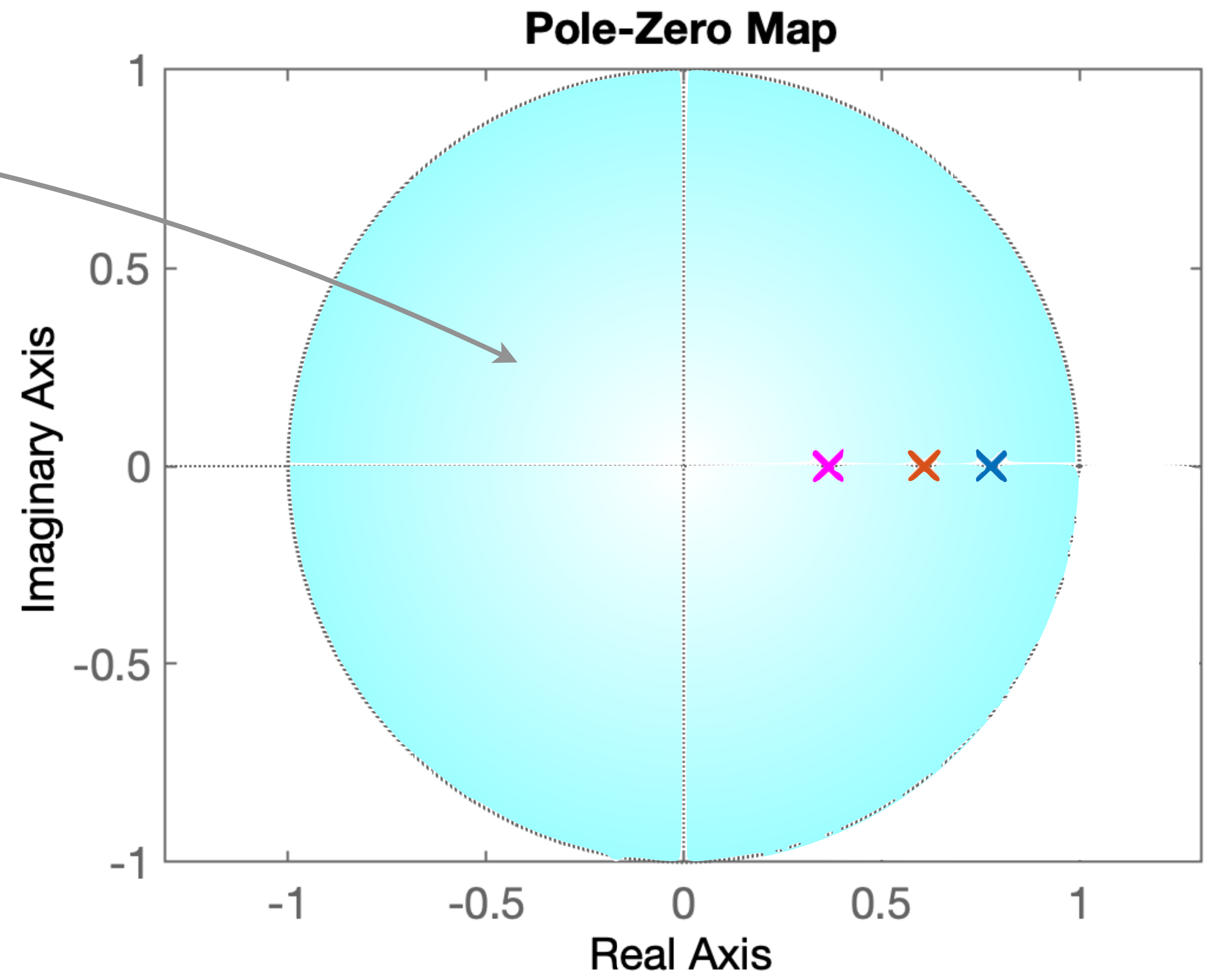
Exemplo:

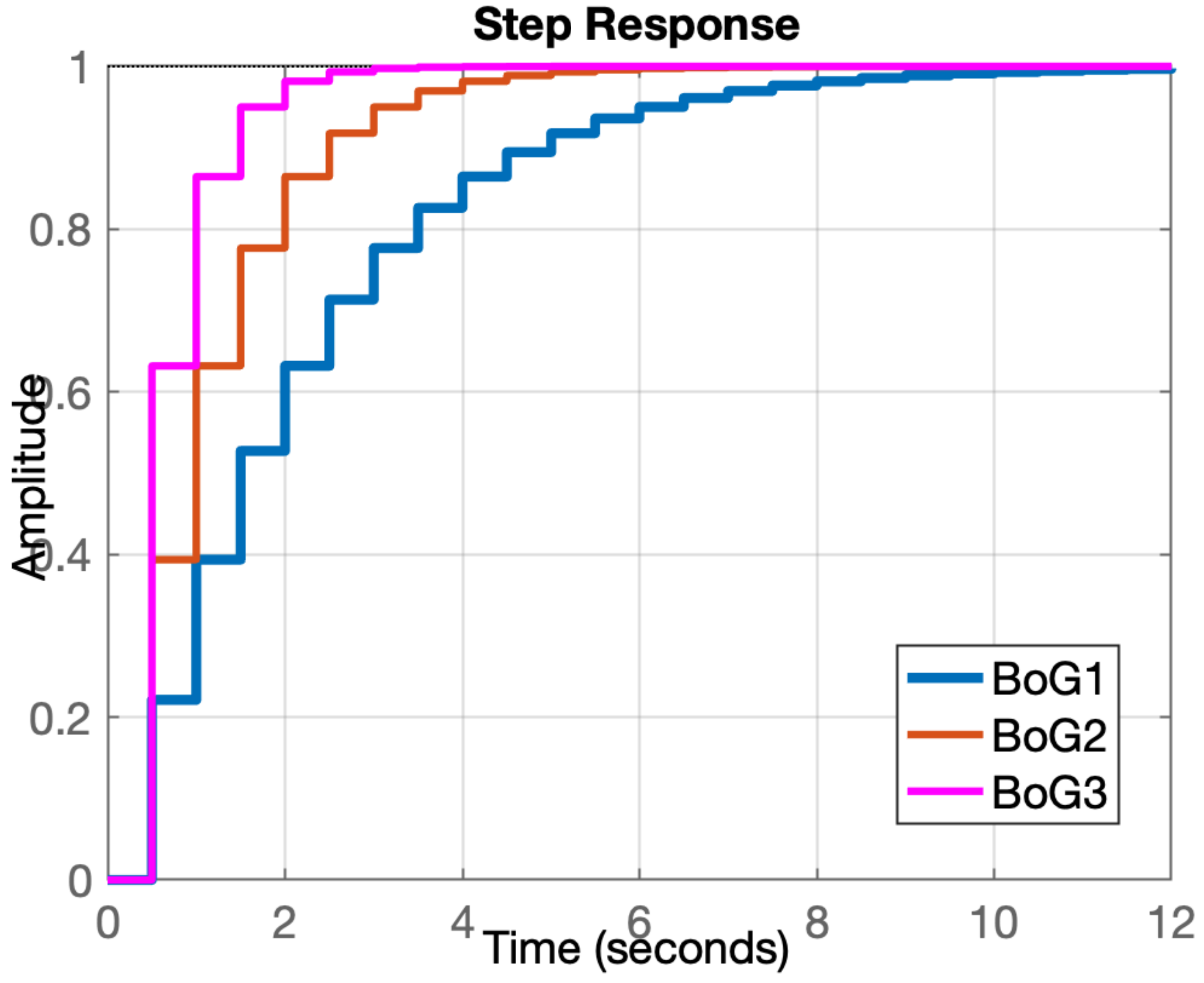
$T = 0,5$ segundos

$$BoG_1(z) = \frac{0,2212}{z - 0,7788}$$

$$BoG_2(z) = \frac{0,39347}{z - 0,6065}$$

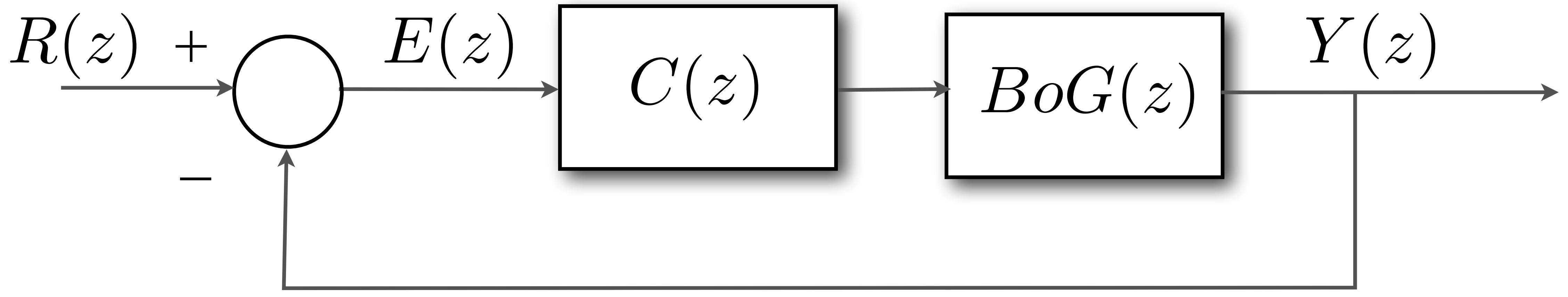
$$BoG_3(z) = \frac{0,63212}{(z - 0,3679)}$$

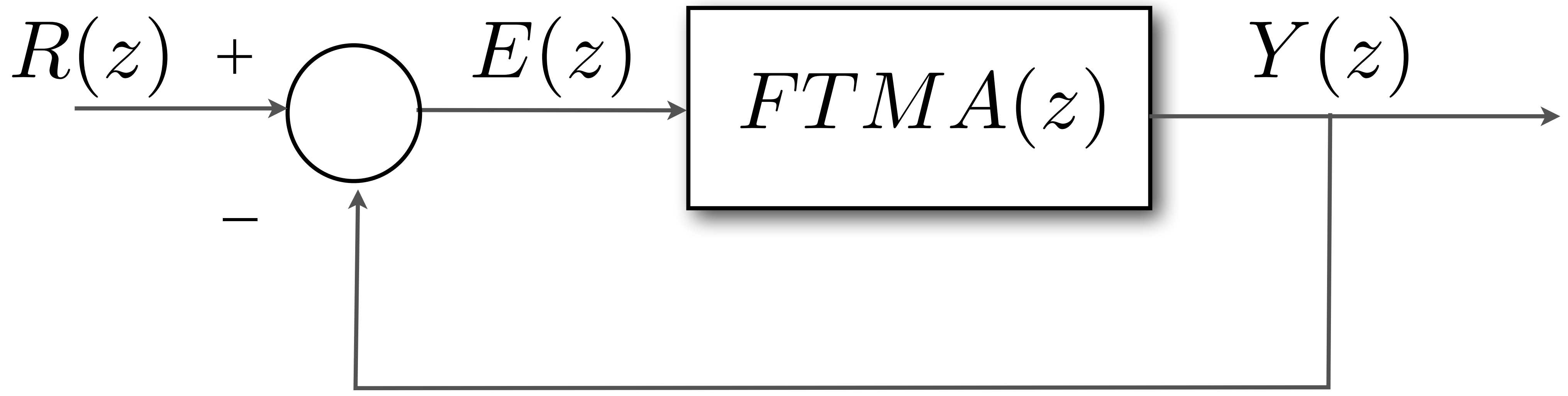


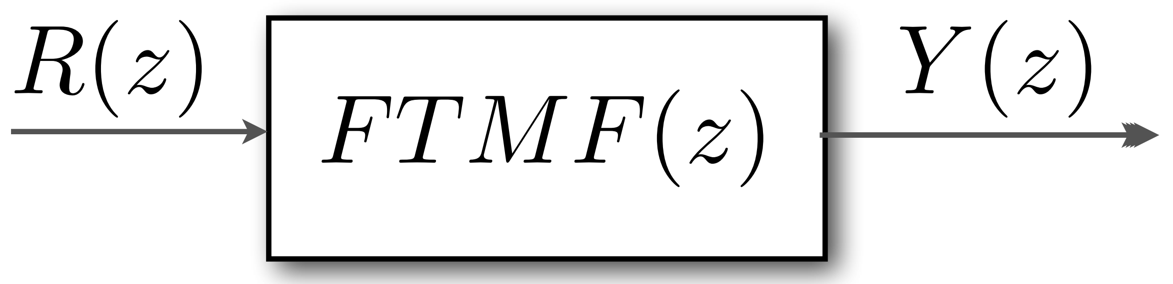


Fechando Malhas de Controle

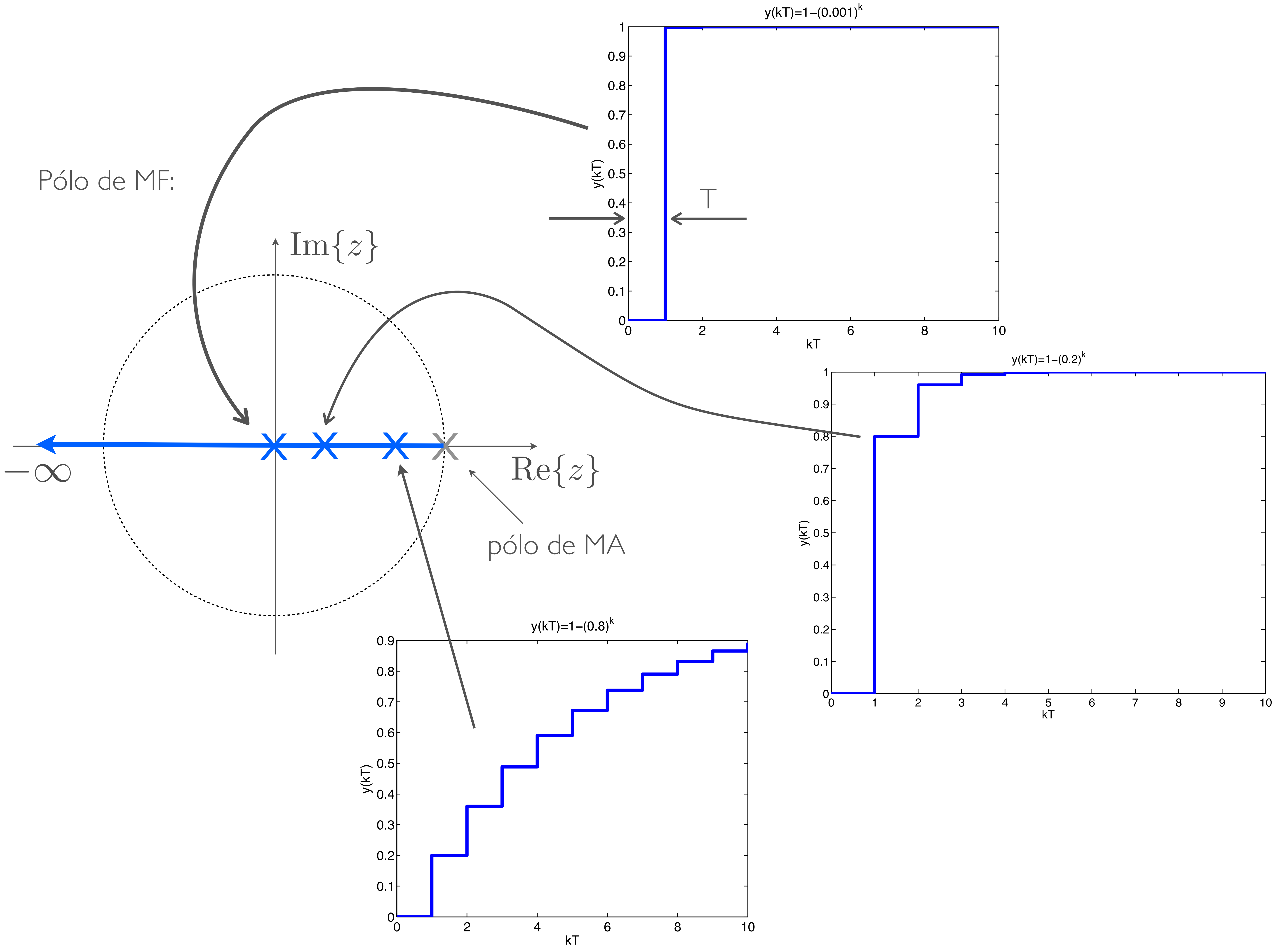
(Da FTMA(z) → FTMF(z))







Respostas de Sistemas
(Entrada Degrau)



Respostas Transientes

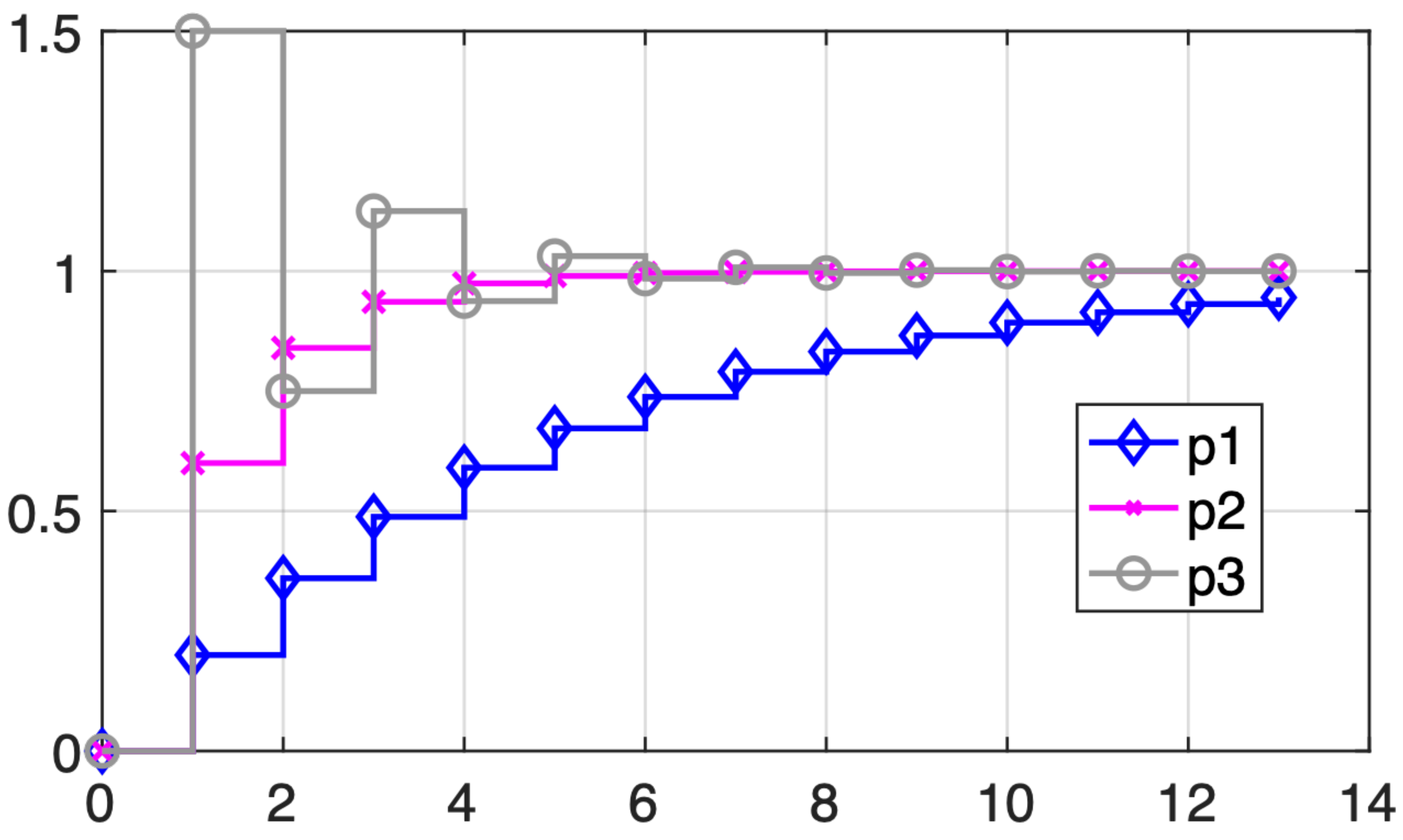
Casos de pólos reais simples

Obs.:

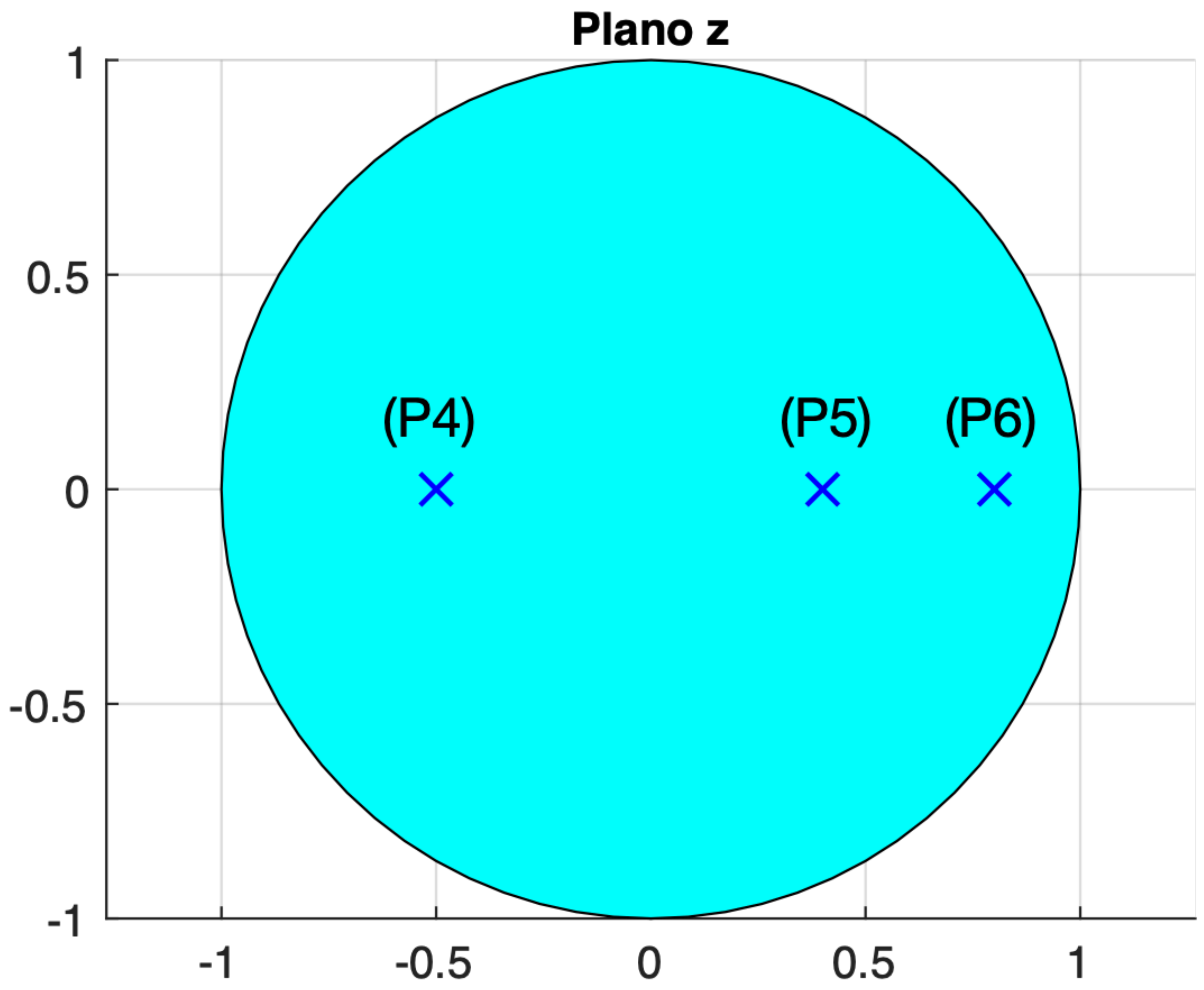
$$\mathbb{Z}\left\{\alpha^k\right\} =$$

$$= \frac{1-\alpha}{z-\alpha}$$

$$(\alpha = e^{-aT})$$



(a) Respostas ao degrau.



(b) Pólos no plano-z.

Respostas Transientes

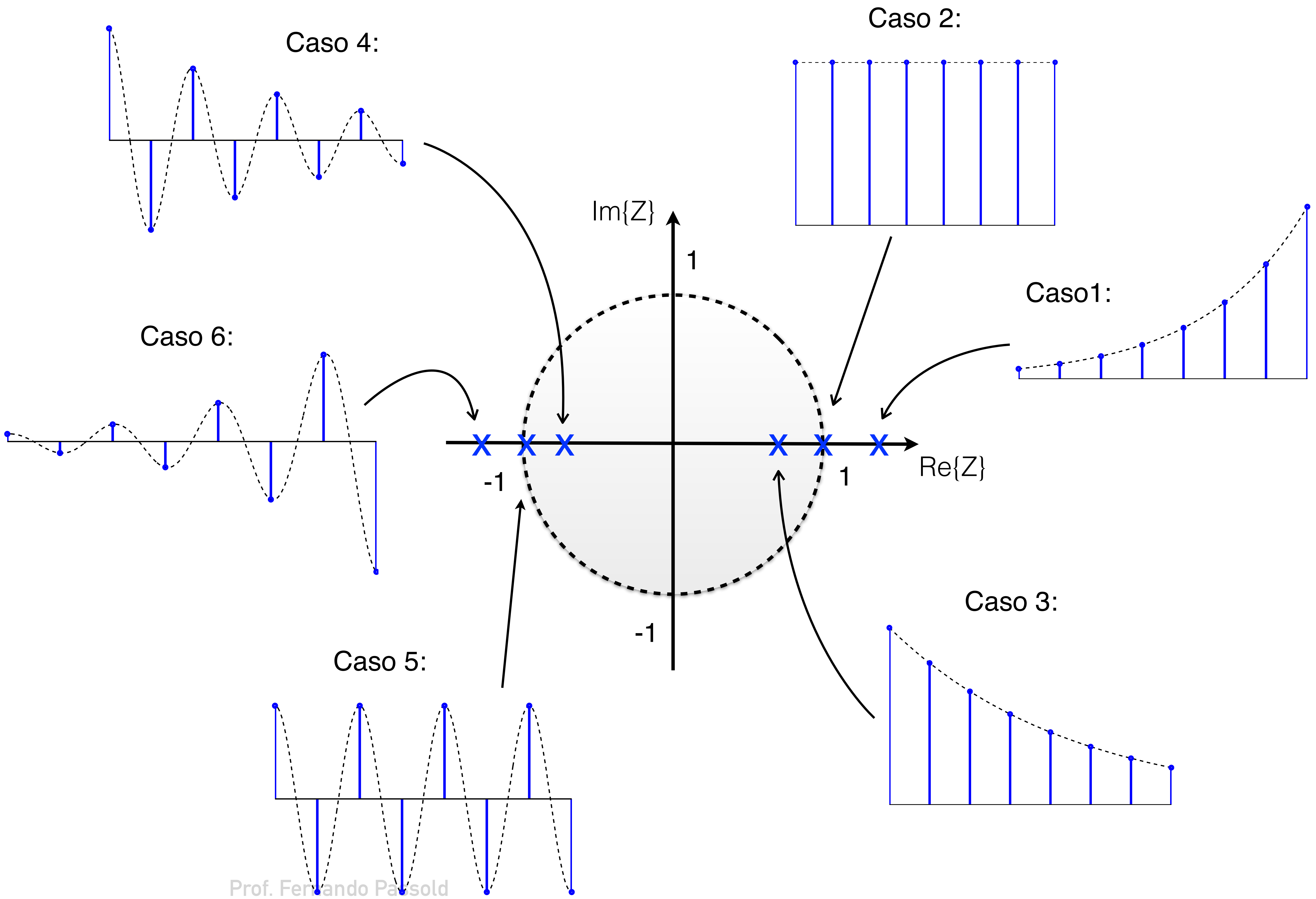
Casos de pólos reais simples

Obs.:

$$\mathbb{Z}\left\{\alpha^k\right\} =$$

$$= \frac{1-\alpha}{z-\alpha}$$

$(\alpha = e^{-aT})$



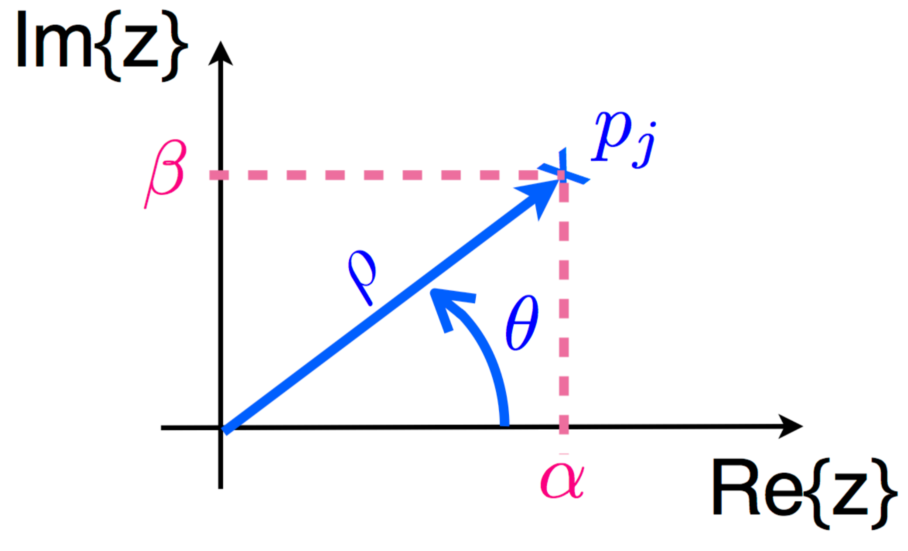
Respostas Transientes

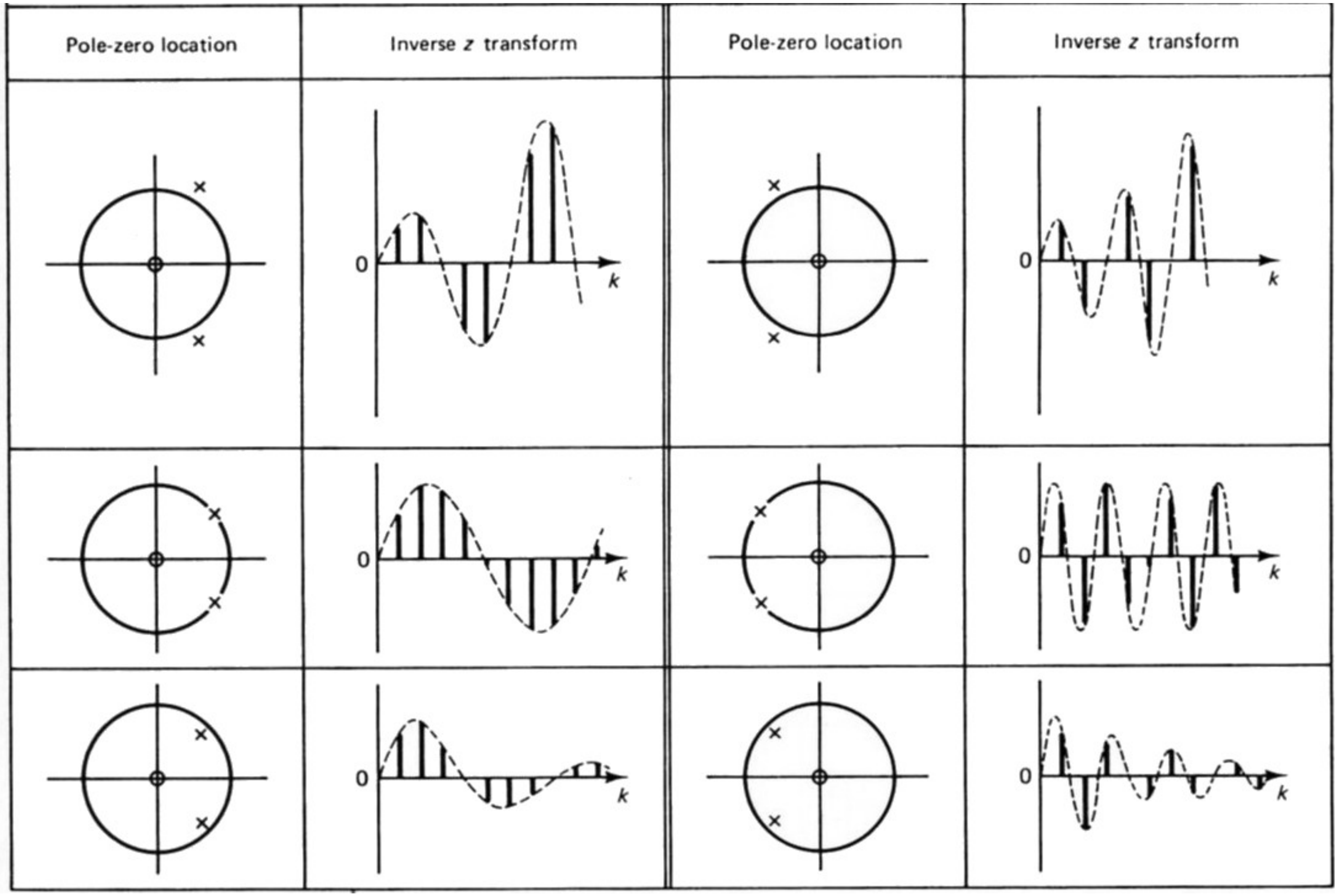
Casos de pólos complexos $(0 < \zeta < 1)$

Obs.:

$$\mathbb{Z}\left\{e^{-akT}\sin(\omega kT)\right\} =$$

$$= \frac{z e^{-aT}\sin(\omega T)}{z^2 - 2e^{-aT}z\cos(\omega T) + e^{-2aT}}$$





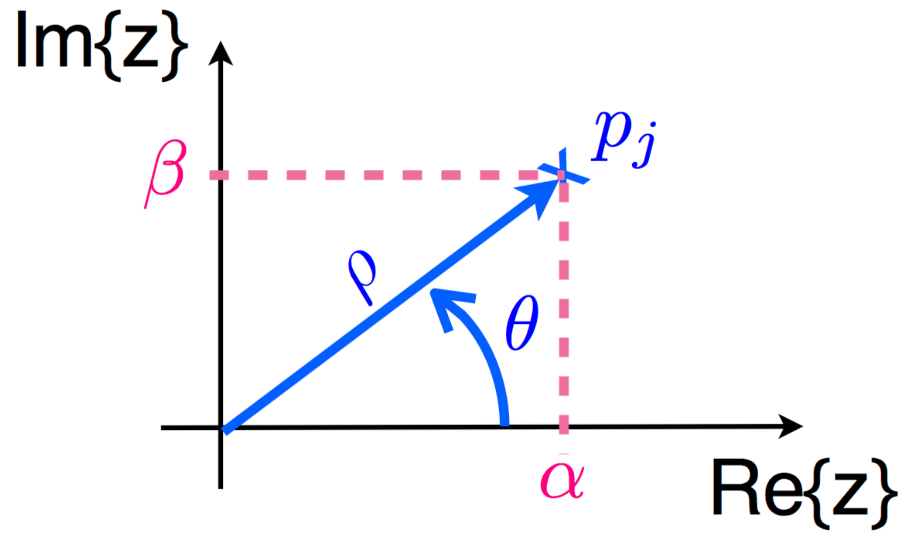
Respostas Transientes

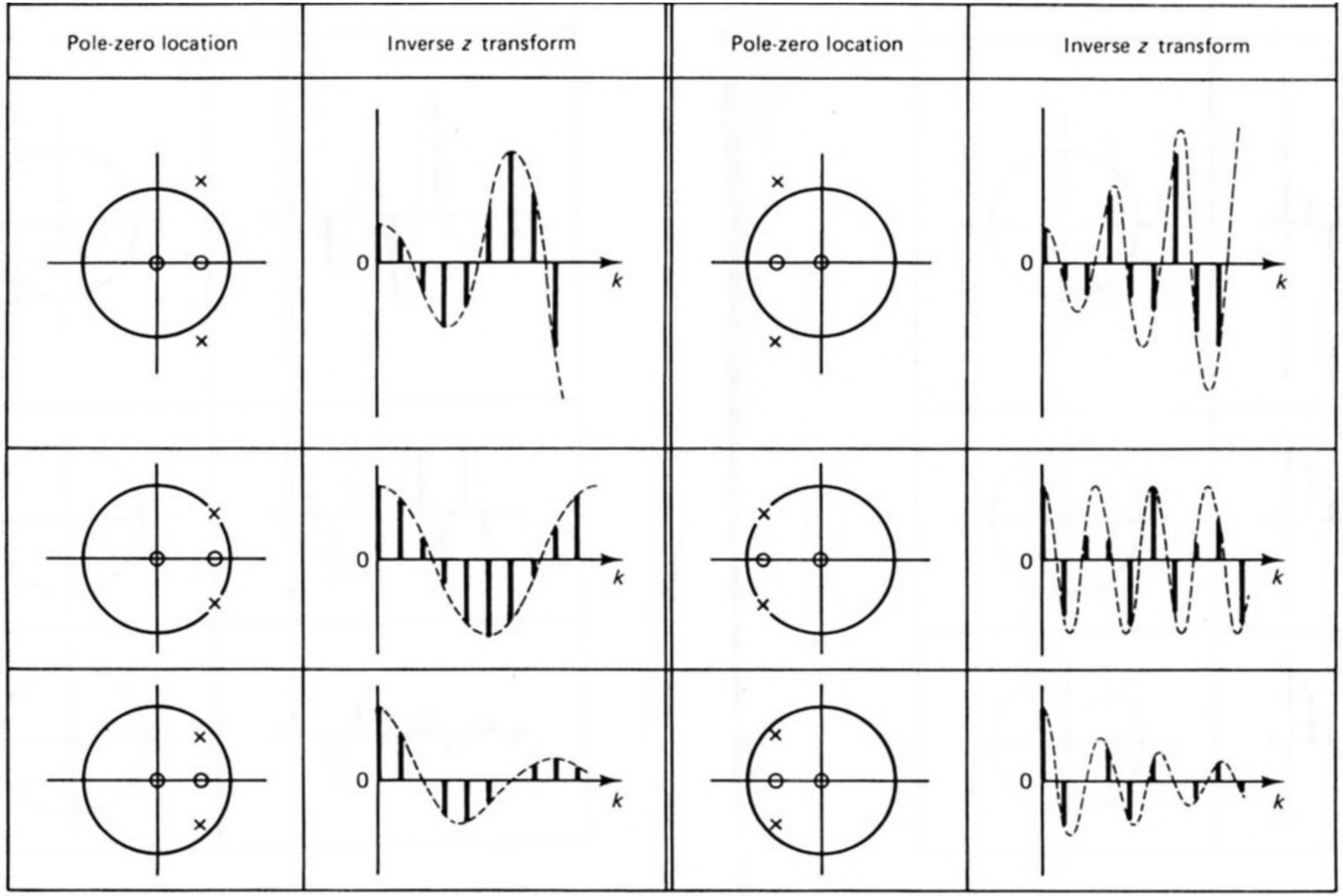
Casos de pólos complexos $(0 < \zeta < 1)$

Obs.:

$$\mathbb{Z}\left\{e^{-akT}\cos(\omega kT)\right\} =$$

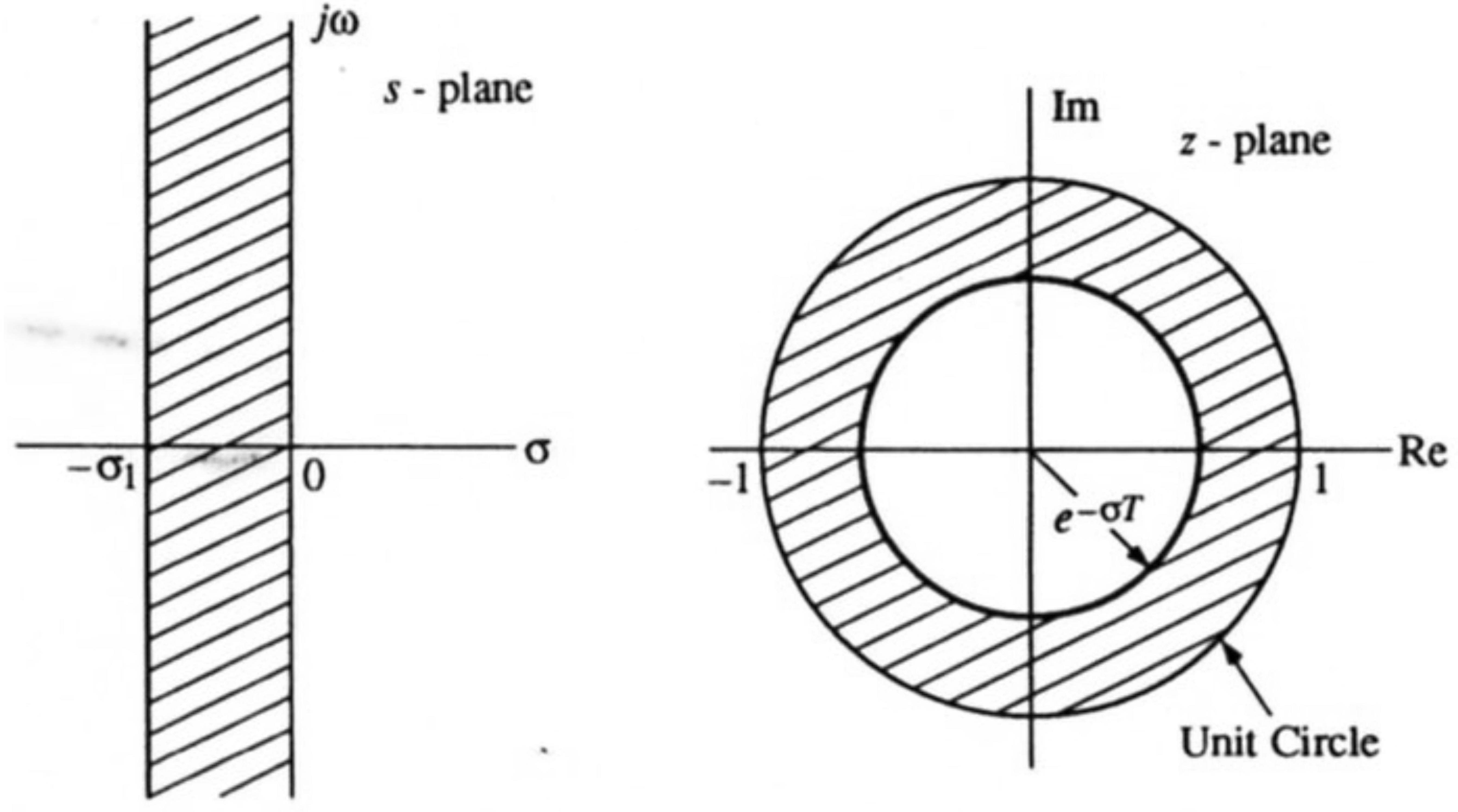
$$= \frac{z[z - e^{-aT}\cos(\omega T)]}{z^2 - 2e^{-aT}z\cos(\omega T) + e^{-2aT}}$$





Paralelos entre plano-s × plano-z

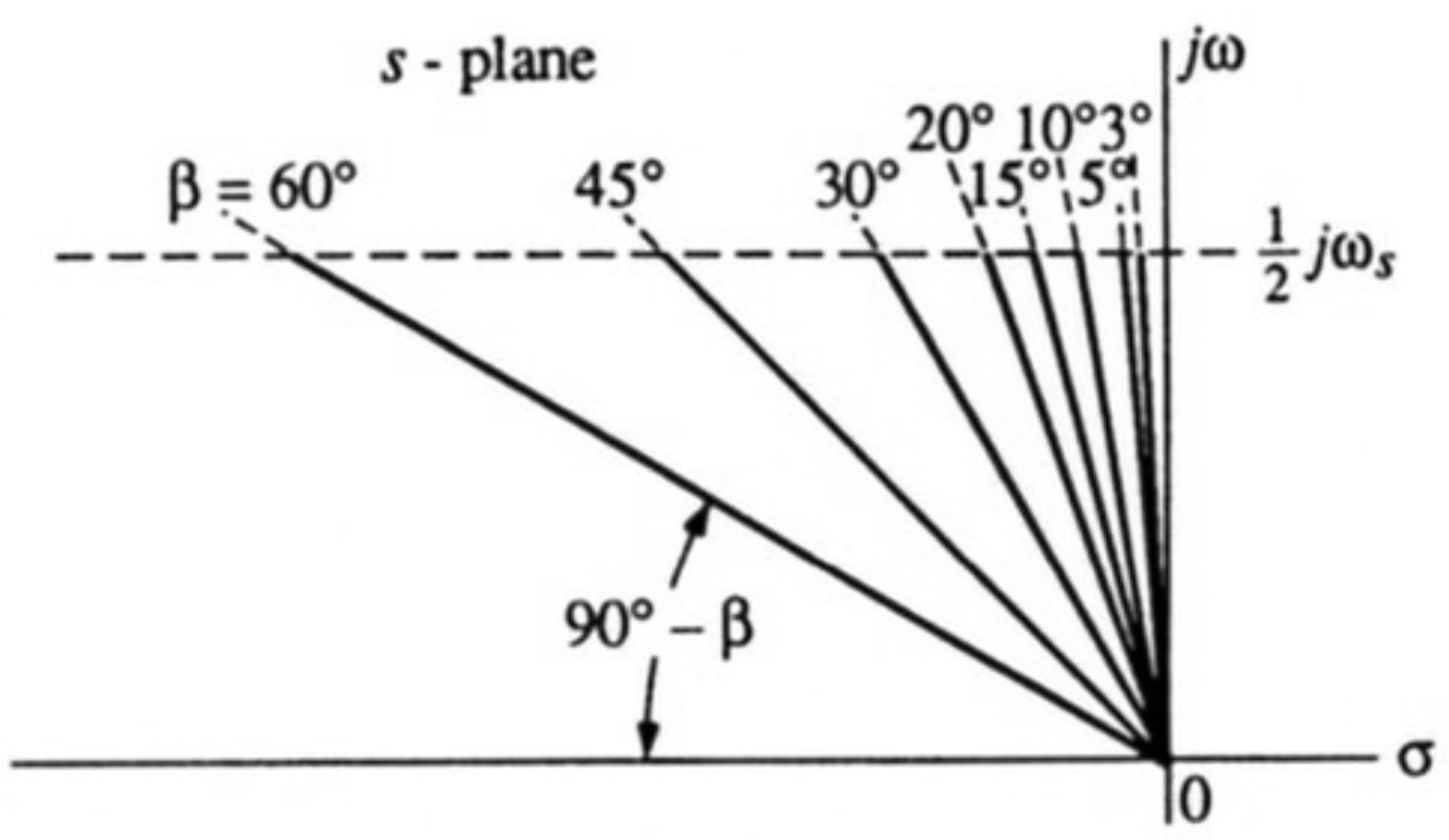
(Mesma parte real - pólos complexos)

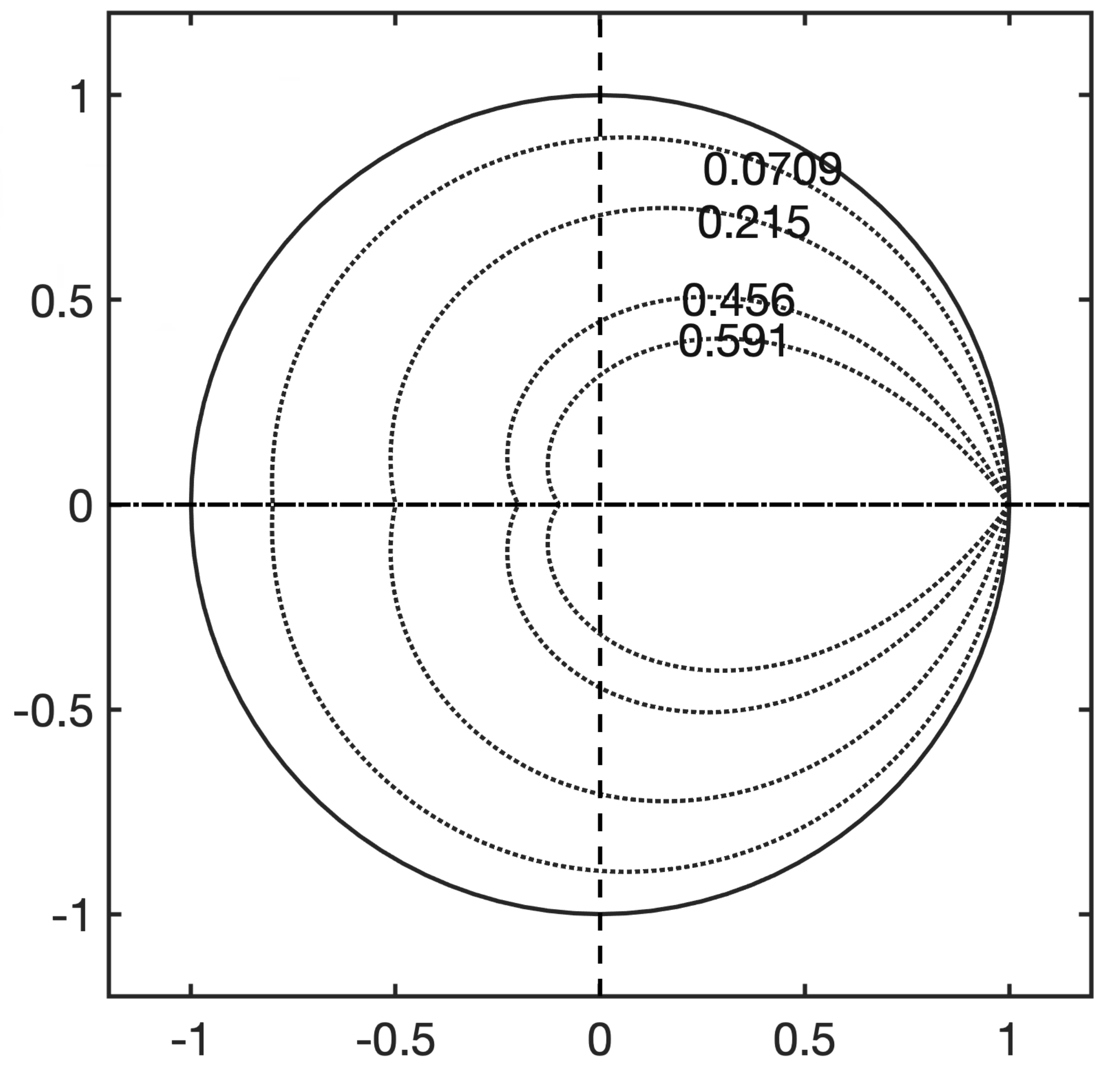


Paralelos entre plano-s × plano-z

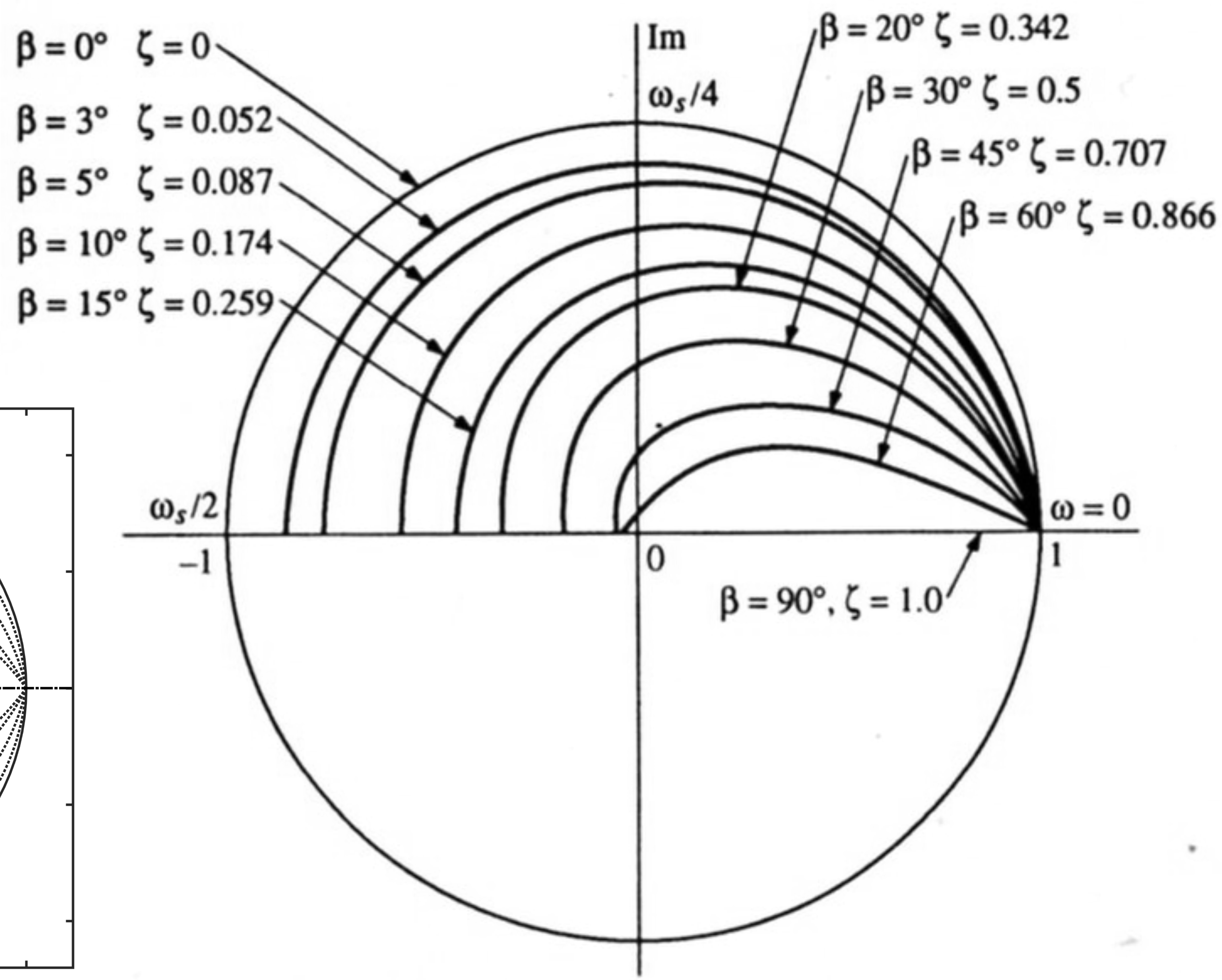
(Mesmo ζ - pólos complexos)

Plano-s



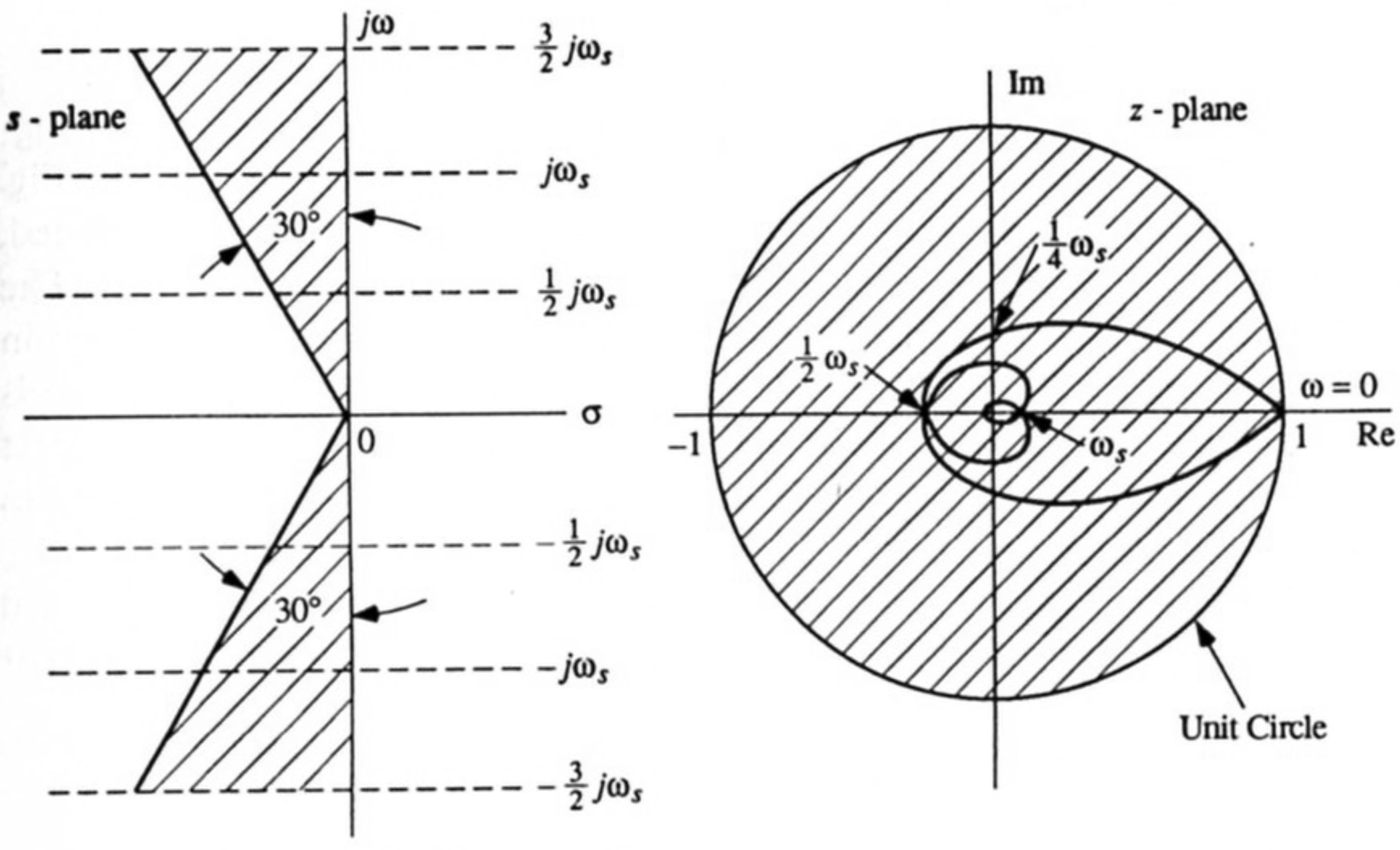


Plano-z

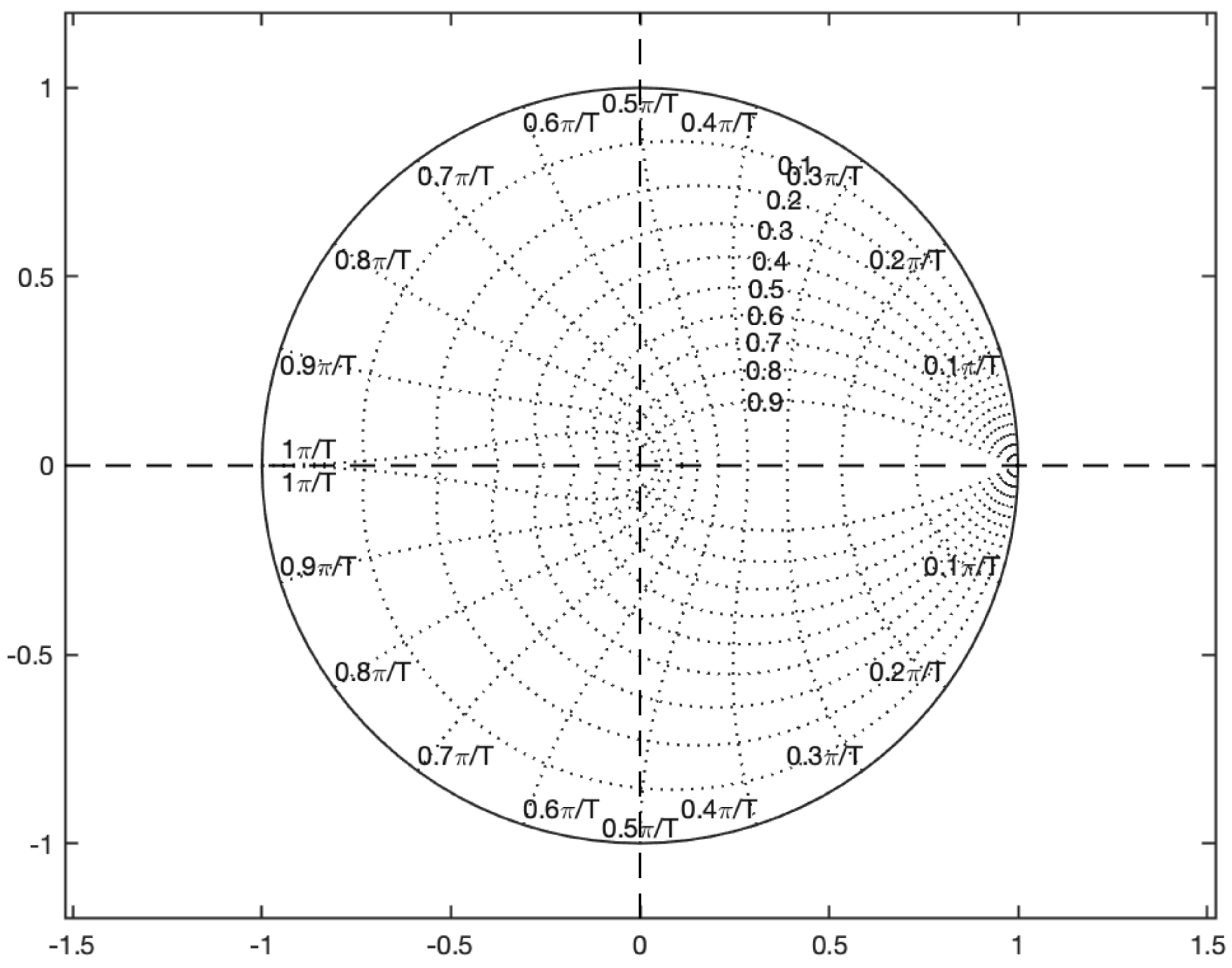


Paralelos entre plano-s $\times$ plano-z

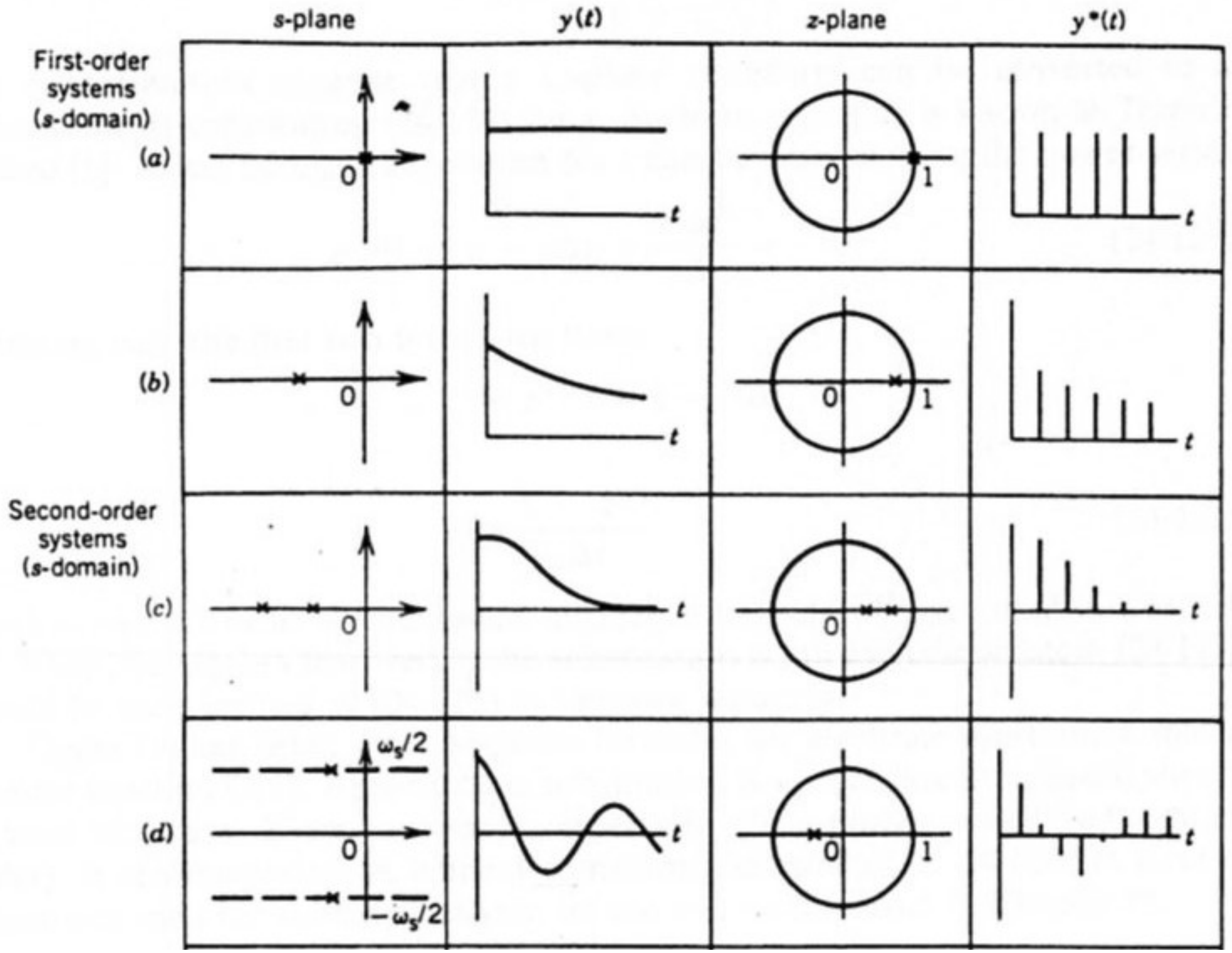
(Mesmo ω_n - pólos complexos)



Plano-z



Paralelos entre plano-s × plano-z



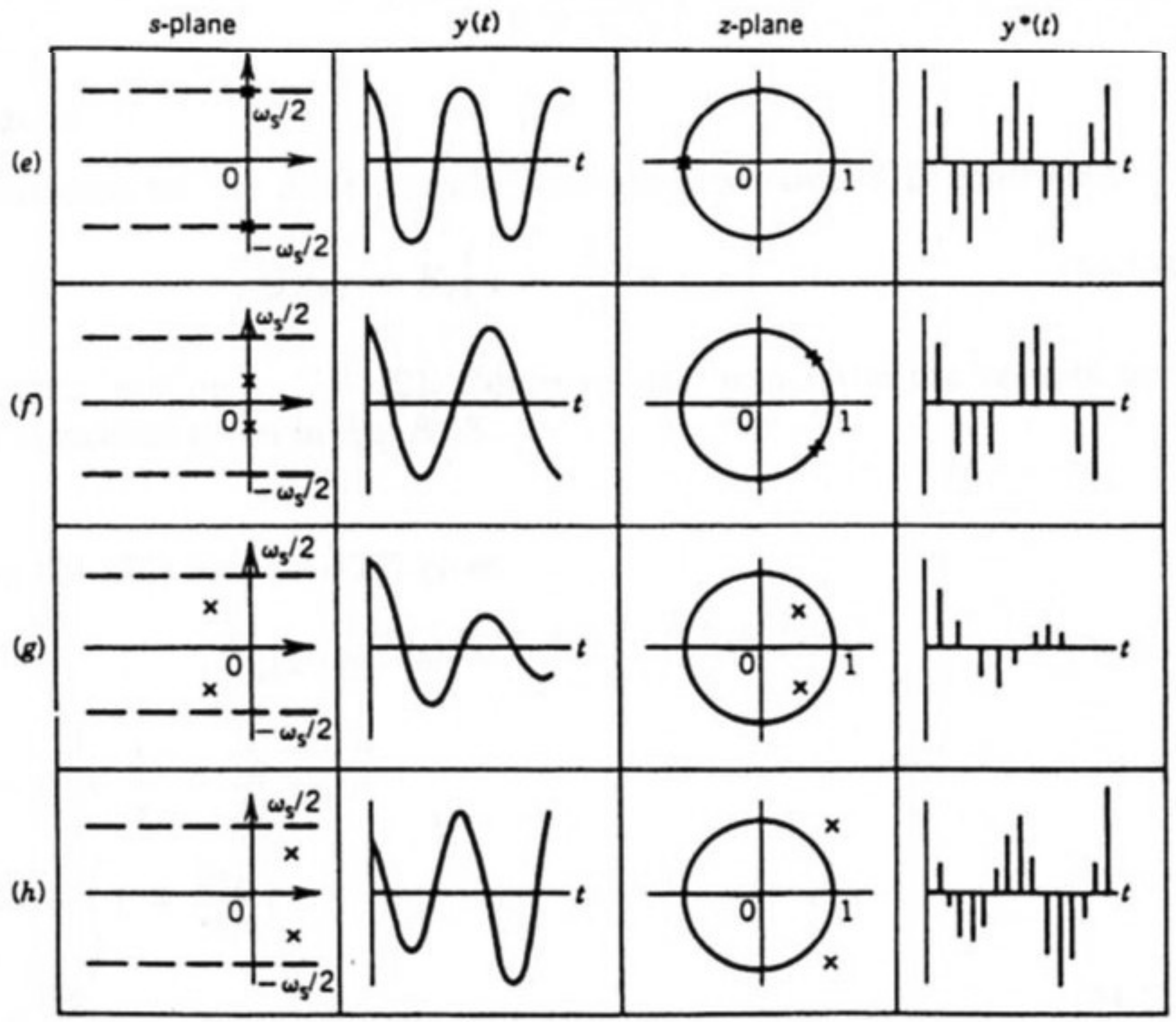
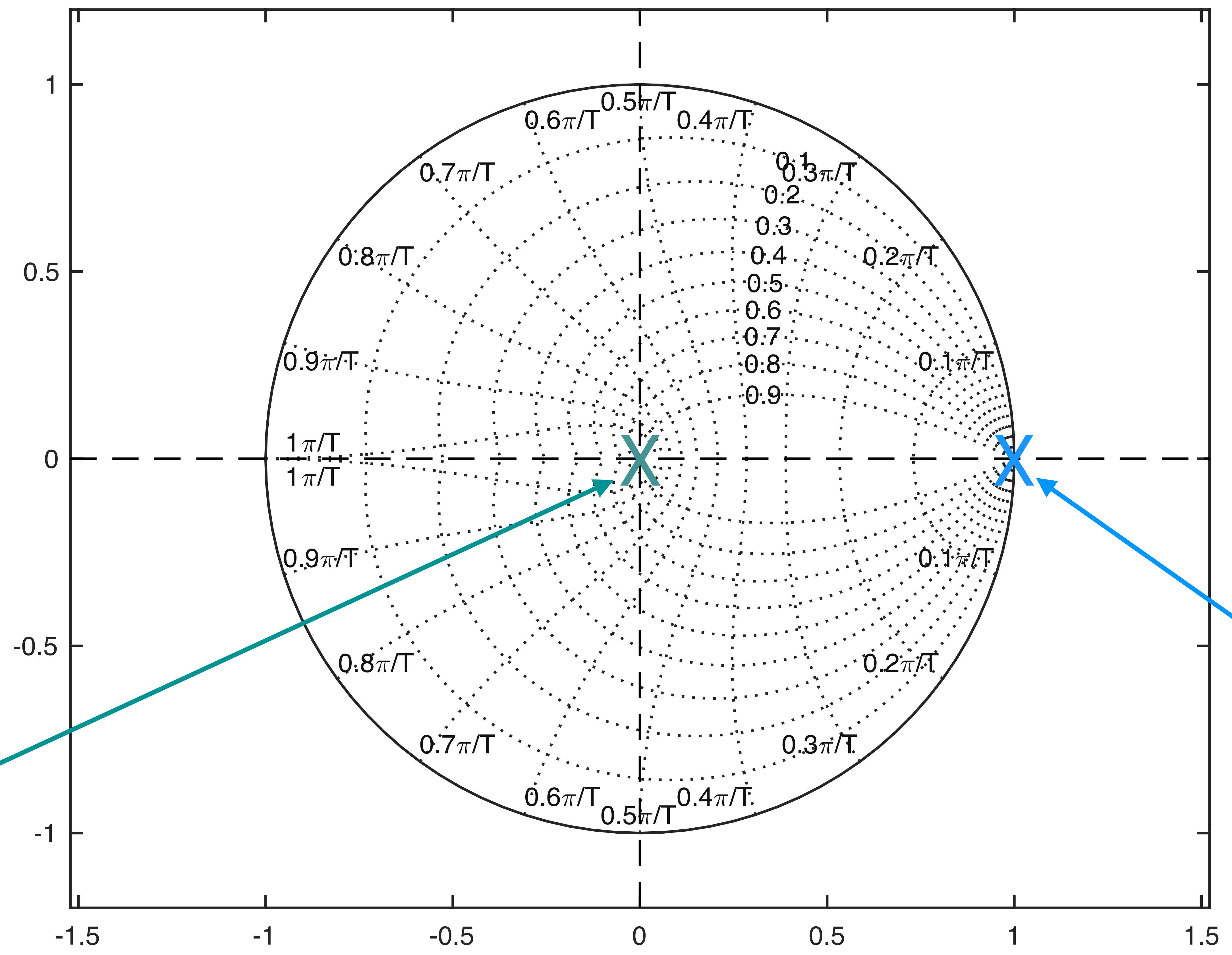


Figure 24.6 Effect of pole locations on impulse response.

Detalhes plano-z

$$z = e^{sT} \quad z^{-1} = e^{-sT}$$



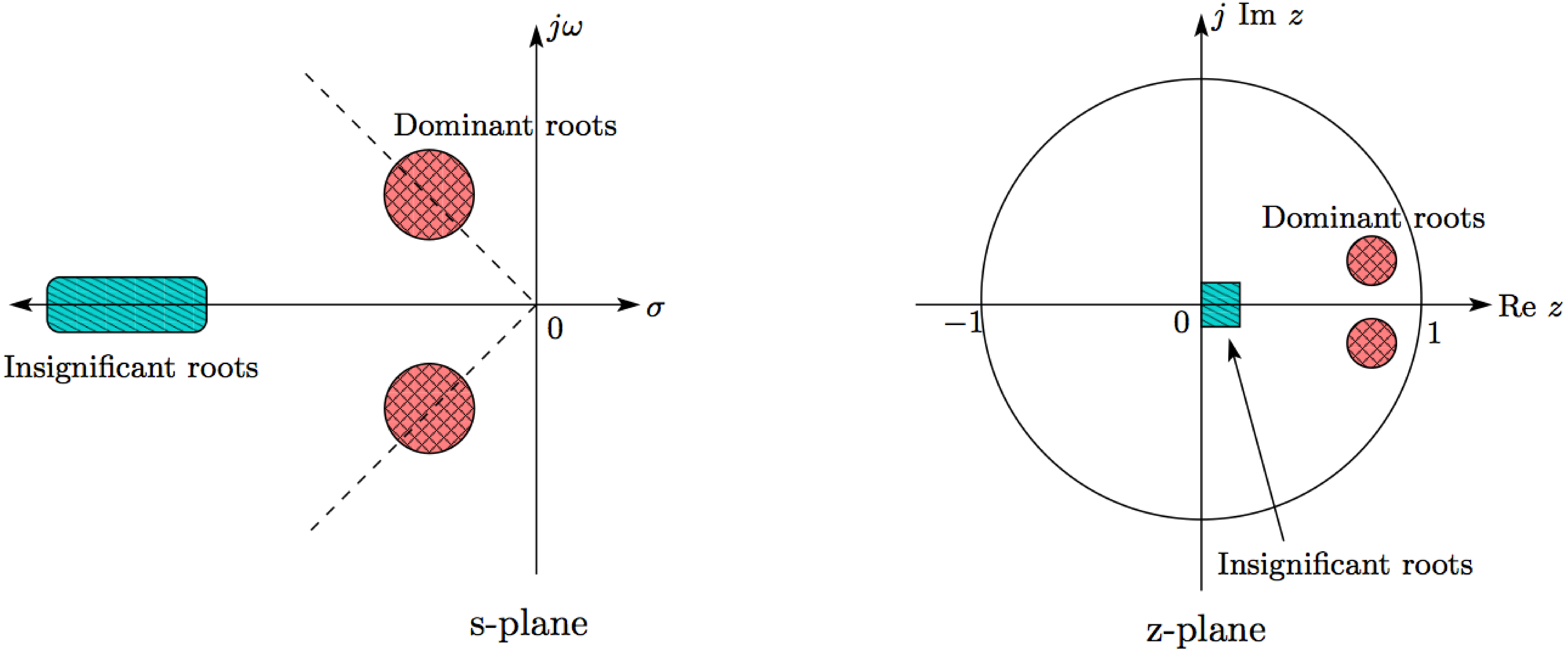
$$T(z) = z^{-1} = \frac{1}{z-0}$$

$$\frac{1}{s} \mapsto T(z) = \frac{z}{z-1}$$

Ref.: ENB458 lecture 3: Welcome to the z-plane, Peter Corke, 17 Spet 2022 [https://youtu.be/zblgvA7Oo-k]

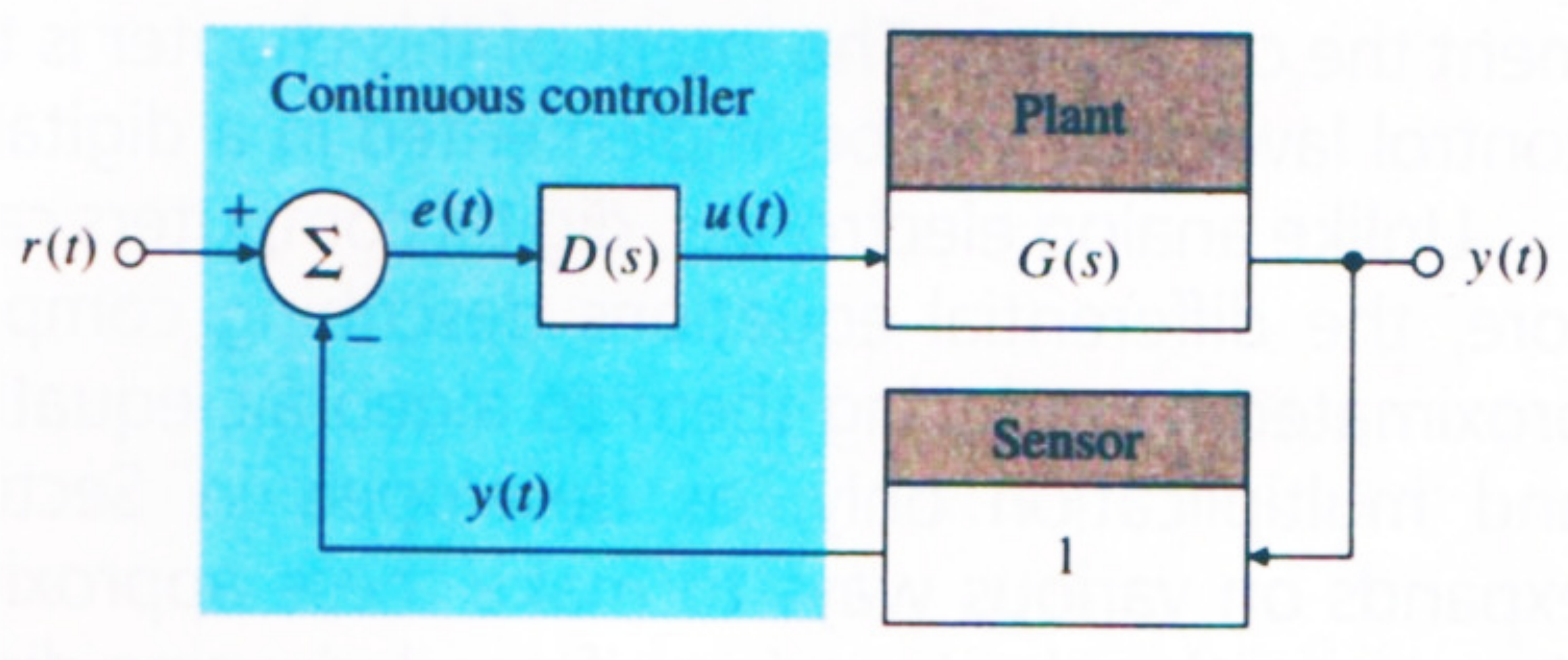
Pólos Dominantes

(No plano-s e no plano-z)

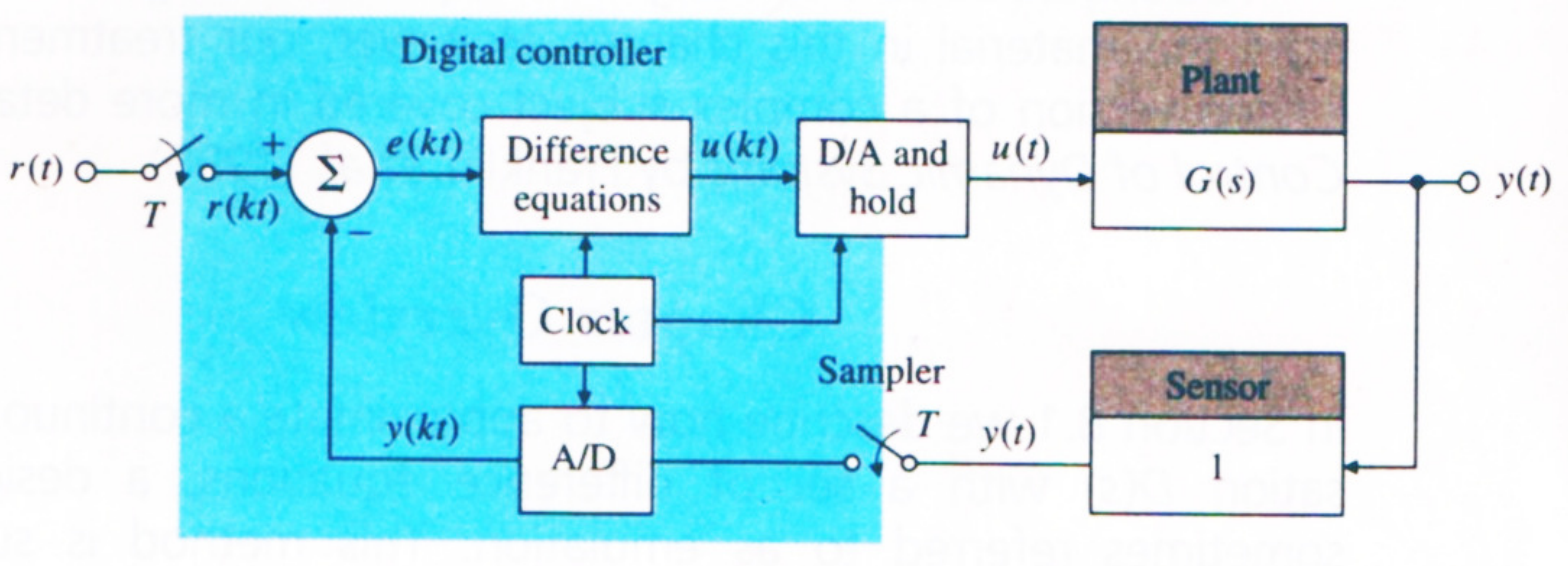


Sistema de Controle
Diagramas de Bloco típicos

Analógico: plano-s



Digital: plano-z

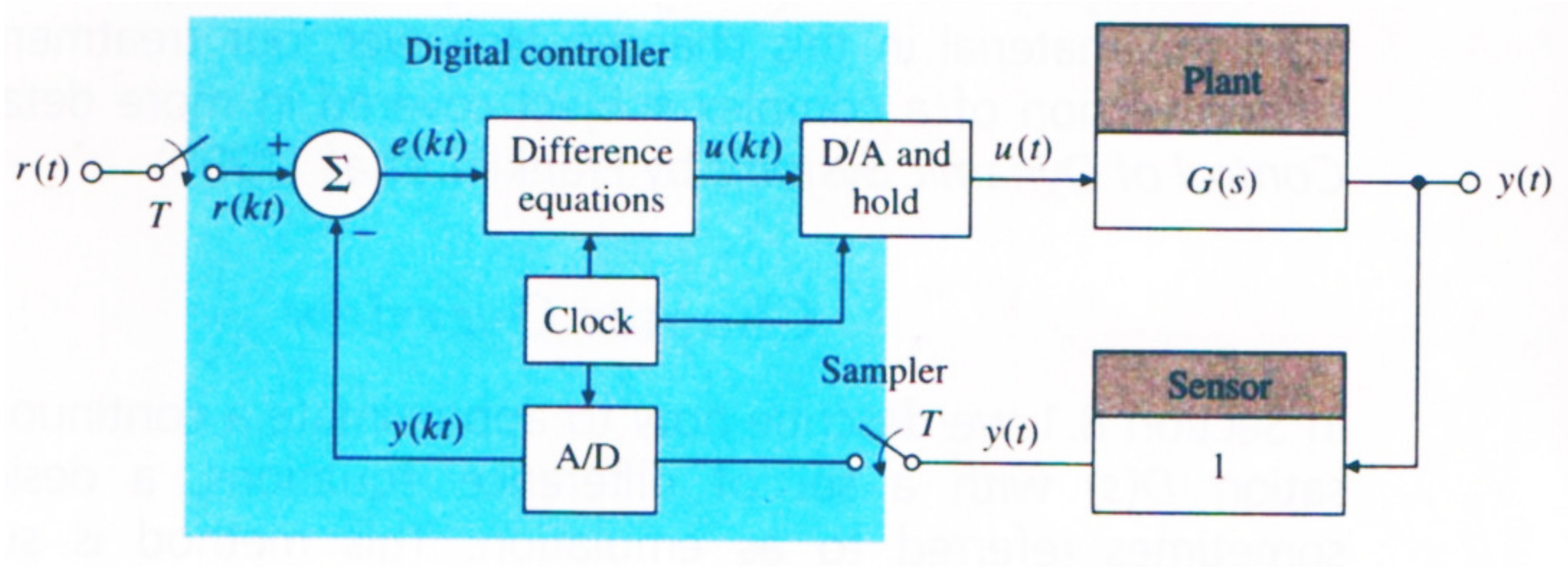


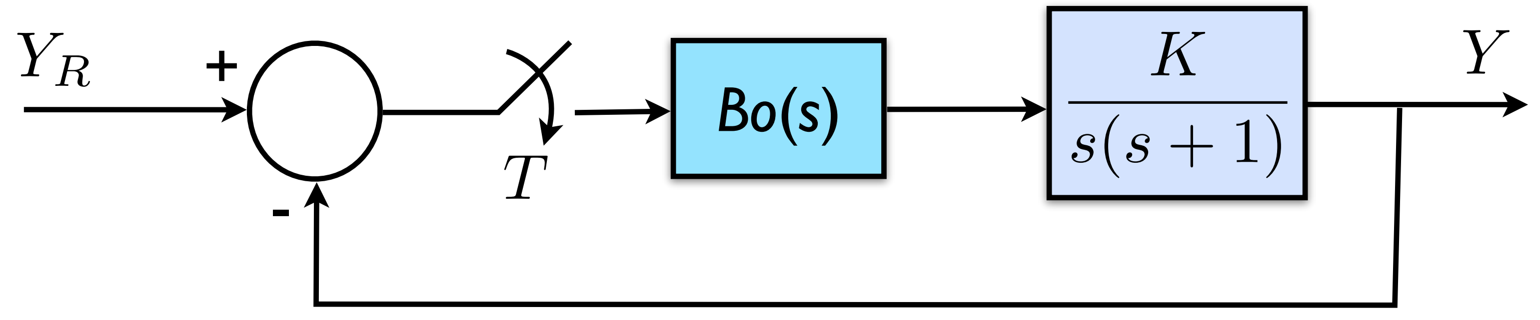
Sistema de Controle

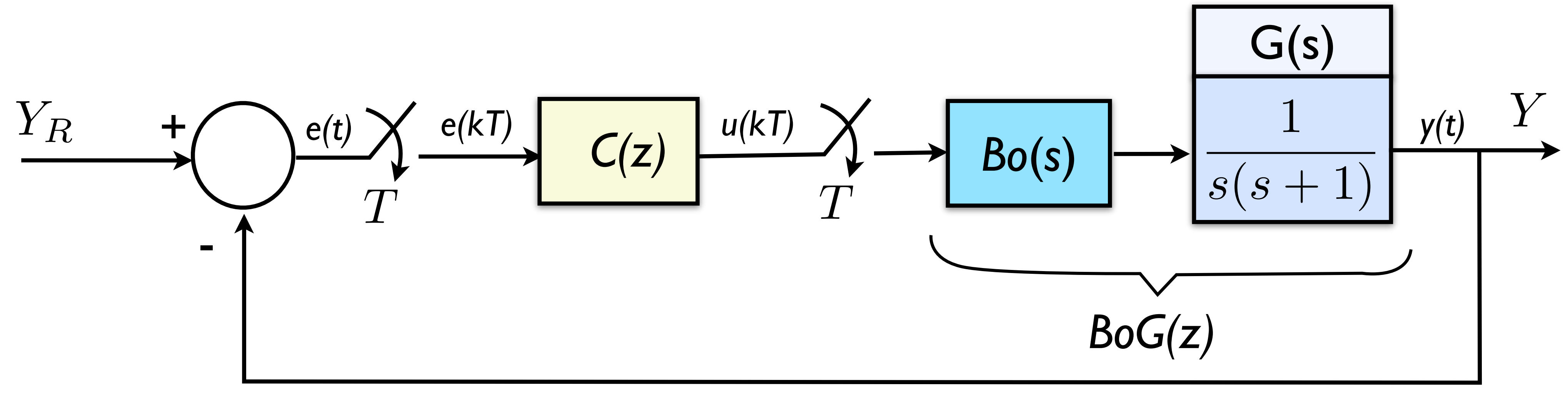
Diagramas de Bloco típicos

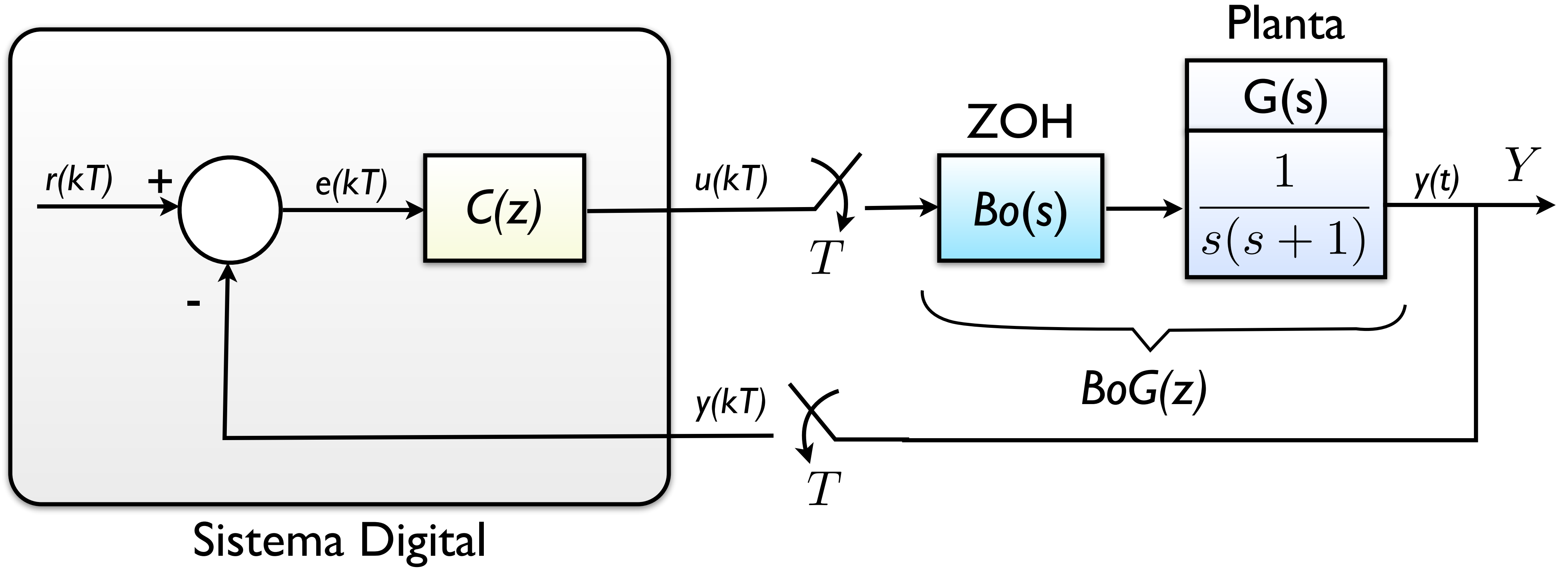
Exemplo:

Digital: plano-z



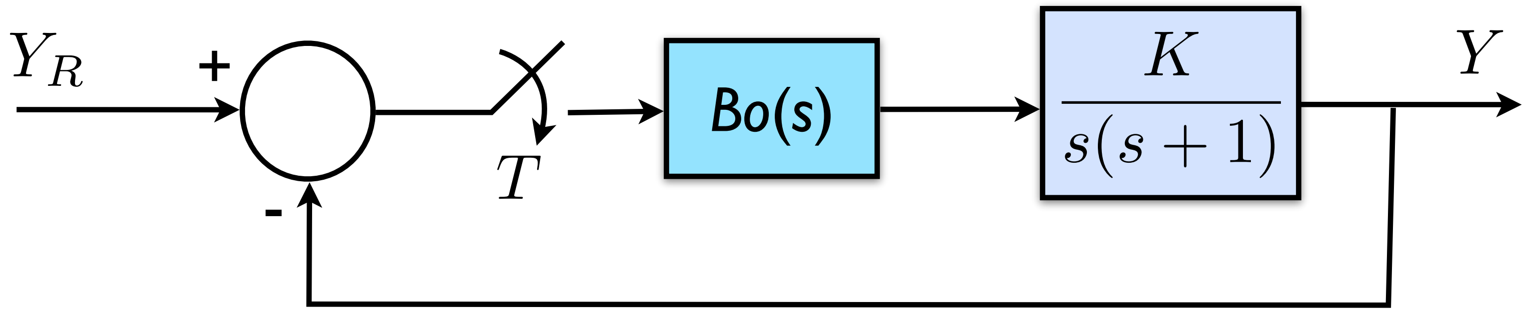


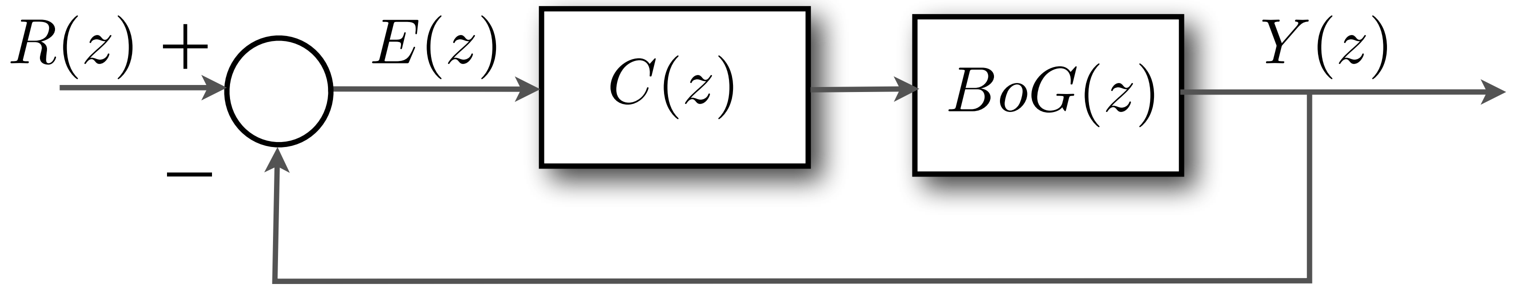


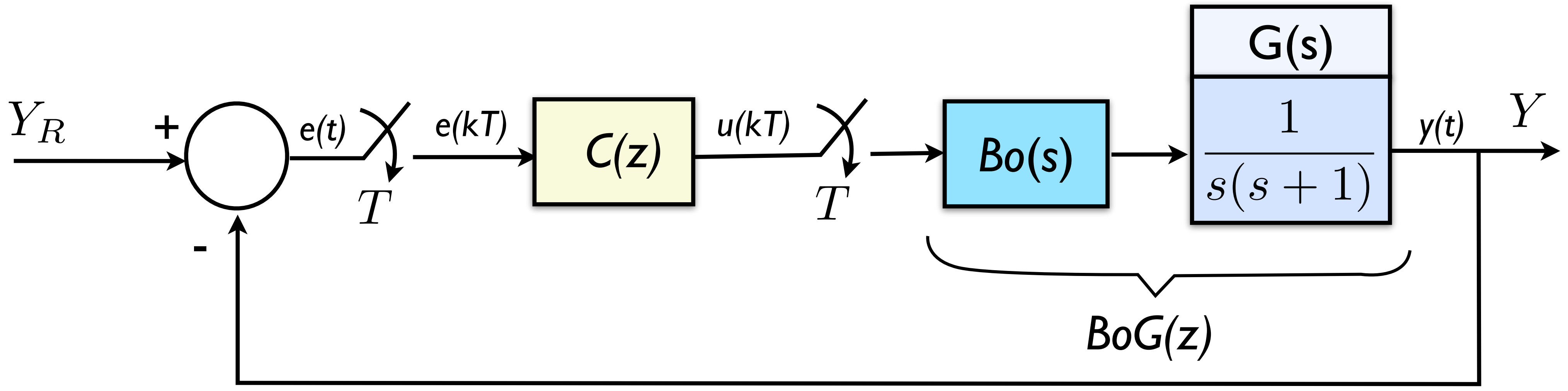


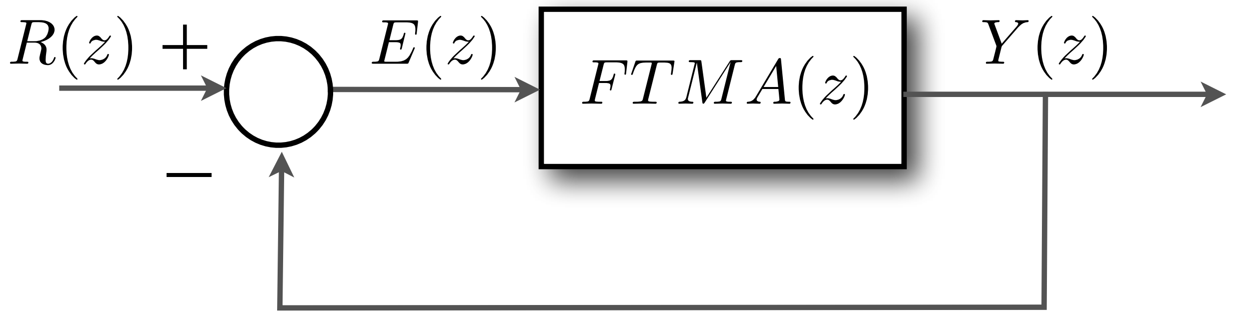
Sistema de Controle
Diagramas de Bloco típicos

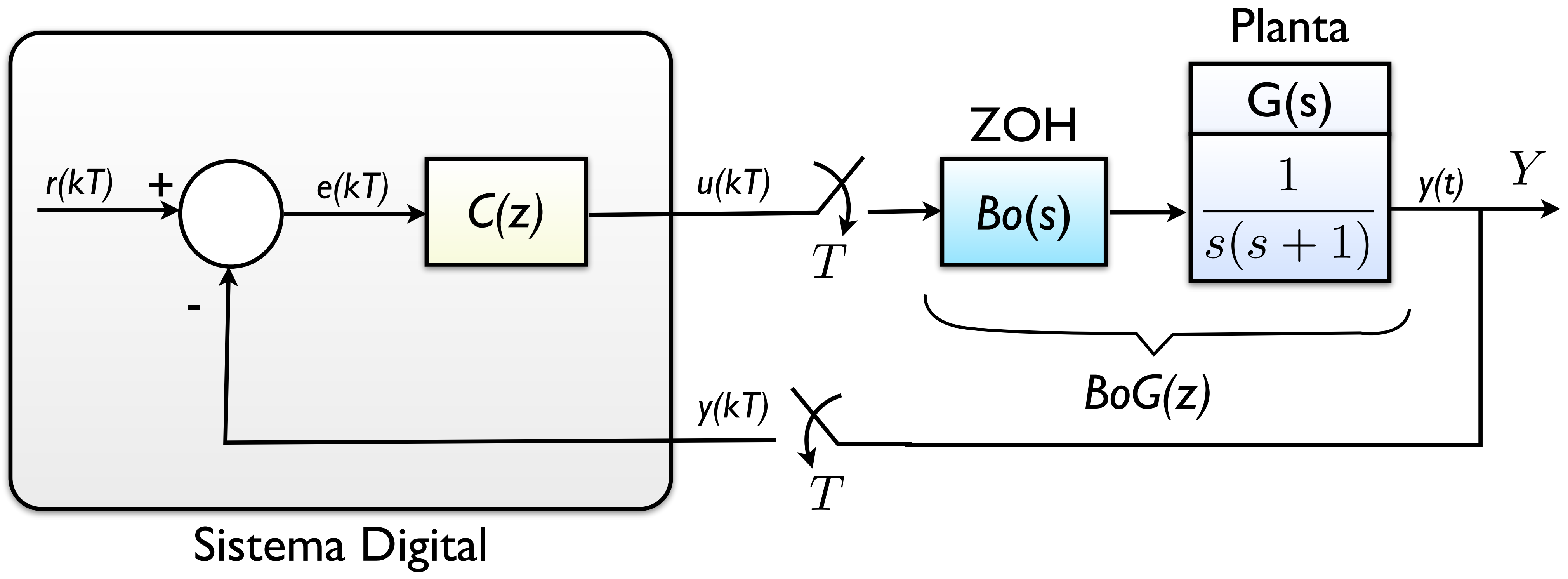
Exemplo:

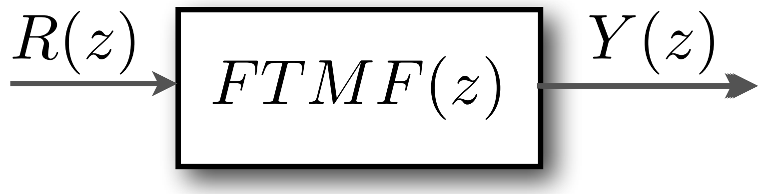








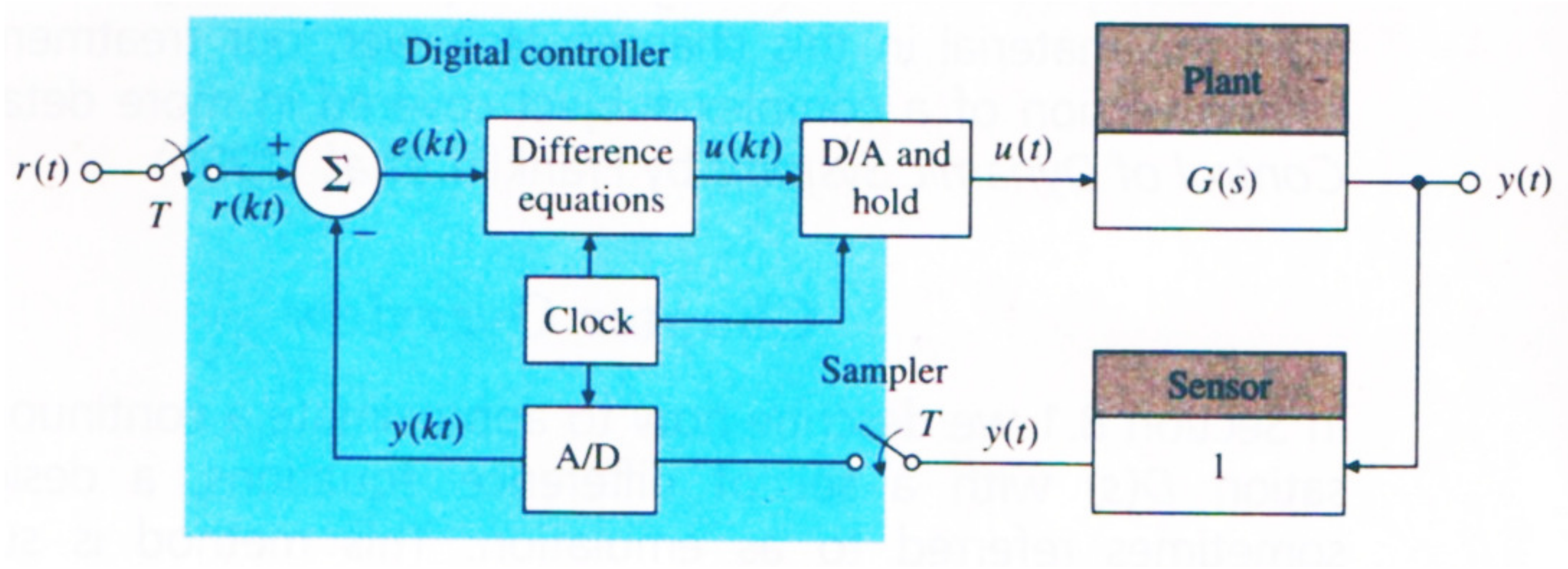


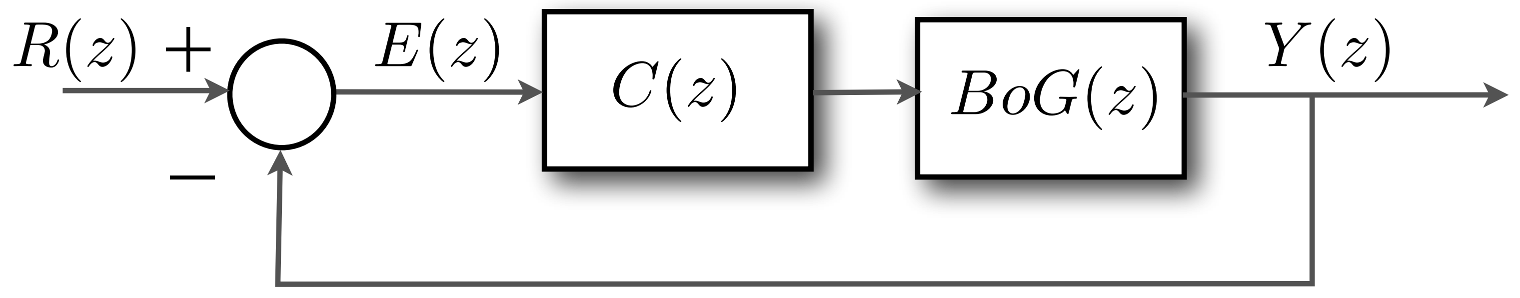


Sistema de Controle

Equação de Diferenças

Digital: plano-z





Exemplo:

$$C(z) = \frac{K(z+a)}{(z+b)}$$

Saída (sinal atuador, de controle): $u[kT] = ?$

$$U(z) = E(z) \cdot C(z)$$

$$C(z) = \frac{U(z)}{E(z)} = \frac{K(z+a)}{(z+b)} \cdot \frac{z^{-1}}{z^{-1}}$$

$$\frac{U(z)}{E(z)} = \frac{K(1+az^{-1})}{(1+bz^{-1})}$$

$$U(z)\left(1+bz^{-1}\right) = E(z)K\left(z+az^{-1}\right)$$

$$U(z) = E(z)K\left(z+az^{-1}\right) - bz^{-1}U(z)$$

Lembrando que: $z^{-1}F(z) \rightleftharpoons f[k-1]$ (uma amostra atrasada):

$$u[k] = K \cdot e[k] + K \cdot a \cdot e[k-1] - b \cdot u[k-1]$$