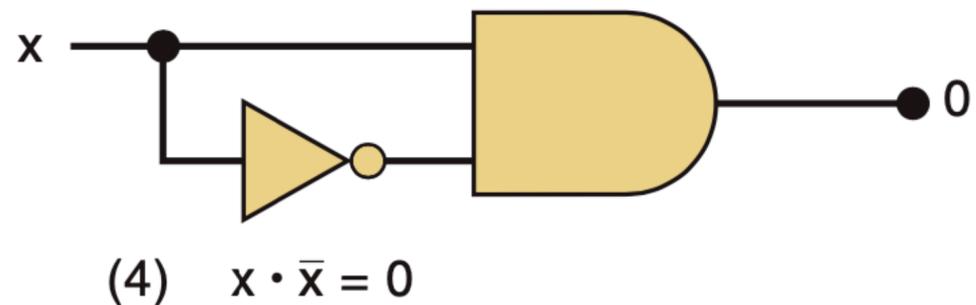
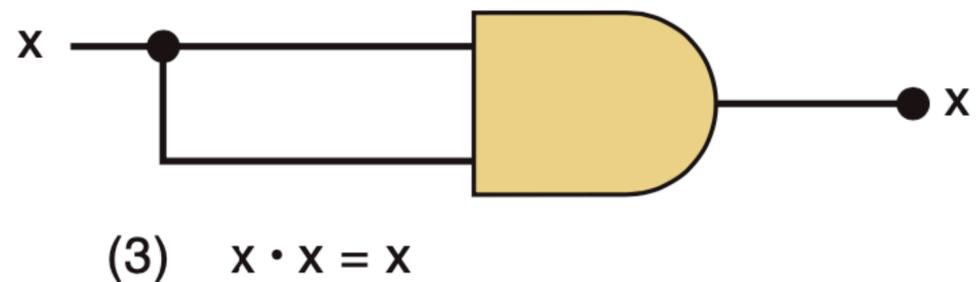
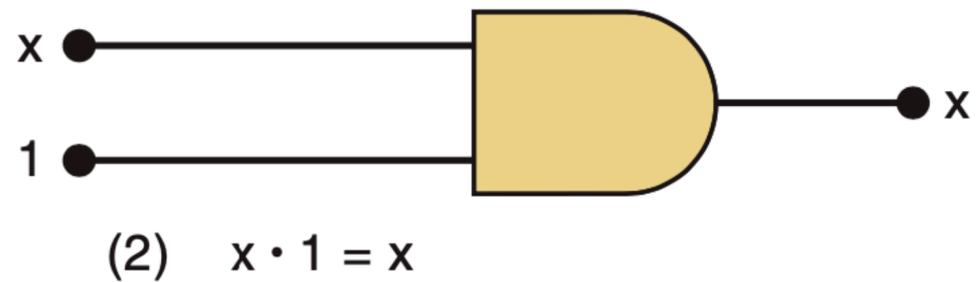
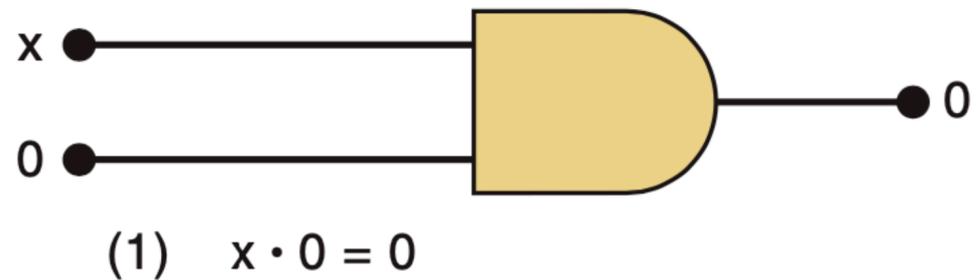


# Álgebra Booleada

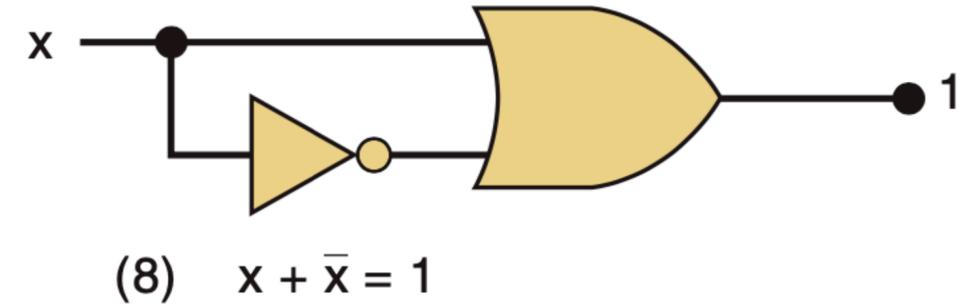
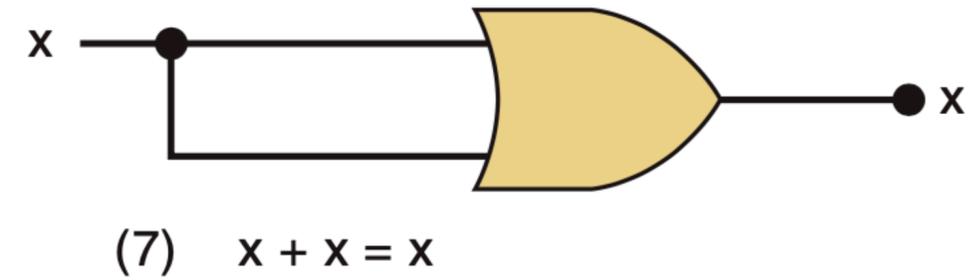
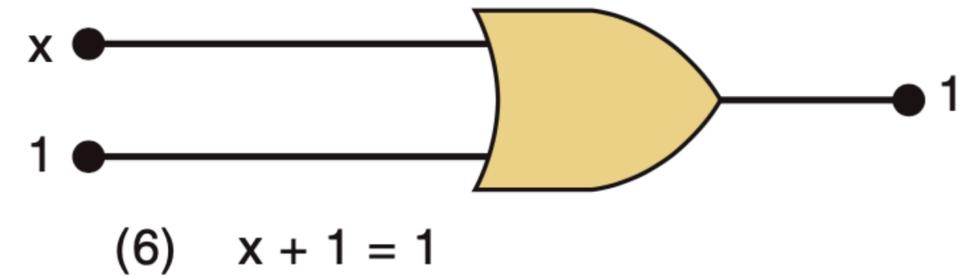
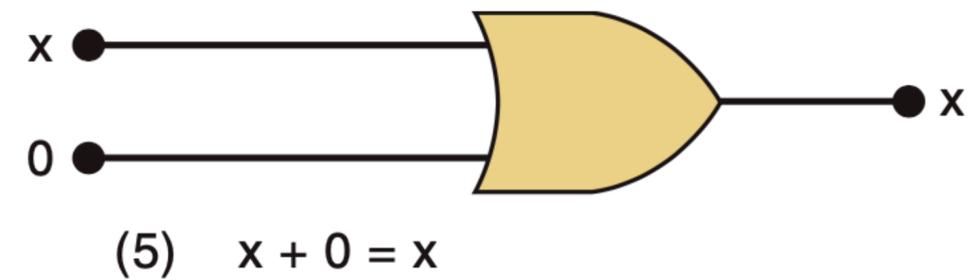
Circuitos Digitais I  
Prof. Fernando Passold

# Teoremas (iniciais)

- Portas AND:



- Portas OR:



# Teoremas Multivariáveis

		Teoremas:	
(9)	$x + y = y + x$	(1) $x \cdot 0 = 0$	Comutativas
(10)	$x \cdot y = y \cdot x$	(2) $x \cdot 1 = x$	
		(4) $x \cdot \bar{x} = 0$	
(11)	$x + (y + z) = (x + y) + z = x + y + z$	(5) $x + 0 = x$	Associativas
(12)	$x(yz) = (xy)z = xyz$	(6) $\bar{x} + 1 = 1$	
		(8) $x + \bar{x} = 1$	
(13a)	$x(y + z) = xy + xz$	(14) $x + xy = x$	Distributivas
(13b)	$(w + x)(y + z) = wy + xy + wz + xz$	(15a) $x + \bar{x}y = x + y$	
		(15b) $\bar{x} + xy = \bar{x} + y$	
(14)	$x + xy = x$		
(15a)	$x + \bar{x}y = x + y$		
(15b)	$\bar{x} + xy = \bar{x} + y$		

Únicas: presentes como material consulta em provas.

# Teoremas Multivariáveis

$$(11) \quad x + (y + z) = (x + y) + z = x + y + z$$

$$(12) \quad x(yz) = (xy)z = xyz$$

Associativas

$$(13a) \quad x(y + z) = xy + xz$$

$$(13b) \quad (w + x)(y + z) = wy + xy + wz + xz$$

Distributivas

$$(14) \quad x + xy = x$$

$$(15a) \quad x + \bar{x}y = x + y$$

$$(15b) \quad \bar{x} + xy = \bar{x} + y$$

x	y	xy	x + xy
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

Ou:

$$\begin{aligned} x + xy &= x(1 + y) \\ &= x \cdot 1 \\ &= x \end{aligned}$$

Note:

$$x + 1 = 1$$

Teoremas:

$$(1) \quad x \cdot 0 = 0$$

$$(2) \quad x \cdot 1 = x$$

$$(4) \quad x \cdot \bar{x} = 0$$

$$(5) \quad x + 0 = x$$

$$(6) \quad x + 1 = 1$$

$$(8) \quad x + \bar{x} = 1$$

$$(14) \quad x + xy = x$$

$$(15a) \quad x + \bar{x}y = x + y$$

$$(15b) \quad \bar{x} + xy = \bar{x} + y$$

# Exemplos de uso:

- Ex\_1:  $y = A \bar{B} D + A \bar{B} \bar{D}$

- Solução:

Notamos termos em comum na expressão ( $A$ ):

Teorema (13): Distributivo): colocamos  $A$  em evidência:

$$y = A \bar{B} (D + \bar{D})$$

Usando Teorema (8):  $x + \bar{x} = 1$ , obtemos:

$$y = A \bar{B} \cdot 1$$

$$y = A \bar{B}$$

Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \bar{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \bar{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \bar{x}y = x + y$

(15b)  $\bar{x} + xy = \bar{x} + y$

# Exemplos de uso:

- Ex\_2:  $z = (\bar{A} + B)(A + B)$
- Solução:  $z = B$

## Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \bar{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \bar{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \bar{x}y = x + y$

(15b)  $\bar{x} + xy = \bar{x} + y$

# Exemplos de uso:

- Ex\_2:  $z = (\bar{A} + B)(A + B)$

- Solução:

Usando teorema 13: distributiva:

$$z = \bar{A} \cdot A + \bar{A} \cdot B + B \cdot A + B \cdot B$$

Usando teorema (4):  $\bar{A} \cdot A = 0$  e teorema (3):  $B \cdot B = B$ :

$$z = 0 + \bar{A} \cdot B + B \cdot A + B = \bar{A}B + AB + B$$

Usando teoremas (2) e (6):

$$z = B(\bar{A} + A + 1)$$

Finalmente:

$$z = B$$

Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \bar{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \bar{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \bar{x}y = x + y$

(15b)  $\bar{x} + xy = \bar{x} + y$

# Problemas

Simplifique:

$$1) y = A\bar{C} + AB\bar{C}$$

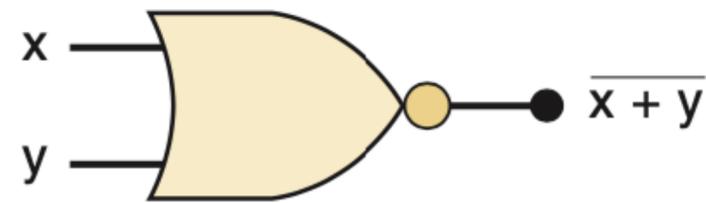
$$2) y = \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$3) y = \bar{A}D + ABD$$

# Teoremas de Demorgan's

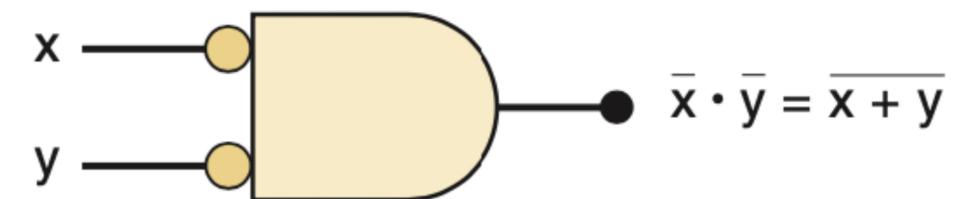
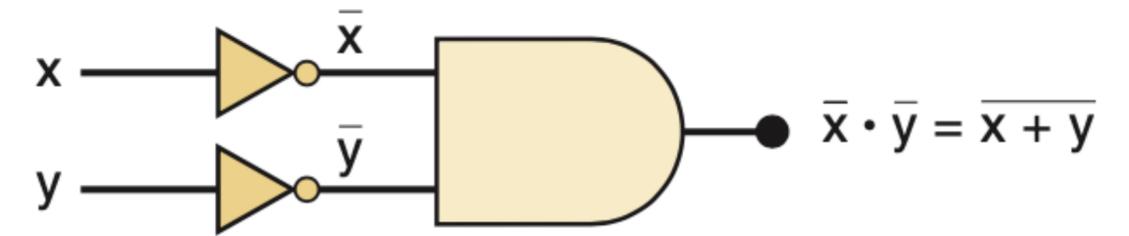
- Associados com portas **NOR** e **NAND** !

$$(16) \overline{(x + y)} = \bar{x} \cdot \bar{y}$$

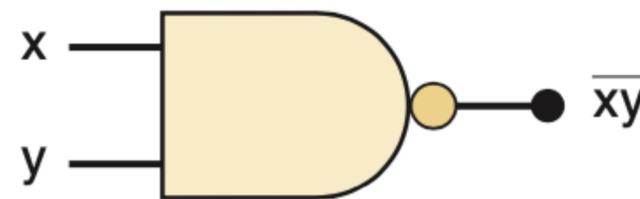


NOR

$\equiv$

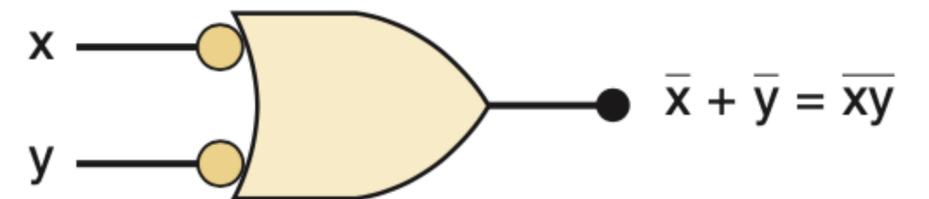
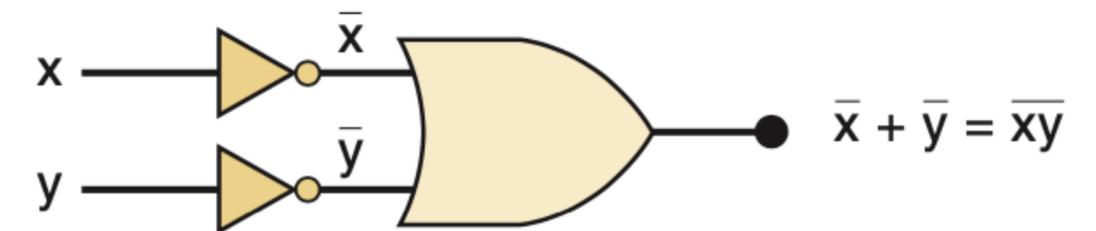


$$(17) \overline{(x \cdot y)} = \bar{x} + \bar{y}$$



NAND

$\equiv$



# Exemplo

- Simplifique a expressão (ou o circuito):

$$z = \overline{(\bar{A} + C) \cdot (B + \bar{D})}$$

- Solução:  $z = A\bar{C} + \bar{B}D$

## Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \bar{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \bar{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \bar{x}y = x + y$

(15b)  $\bar{x} + xy = \bar{x} + y$

(16)  $\overline{(x + y)} = \bar{x} \cdot \bar{y}$

(17)  $\overline{(x \cdot y)} = \bar{x} + \bar{y}$

# Exemplo

- Simplifique a expressão (ou o circuito):

$$z = \overline{(\bar{A} + C) \cdot (B + \bar{D})}$$

- Solução:

$$z = \overline{(\bar{A} + C)} + \overline{(B + \bar{D})}$$

Teorema (17)

$$z = (\bar{A} \cdot \bar{C}) + (\bar{B} \cdot \bar{\bar{D}})$$

Teorema (16)

$$z = A \bar{C} + \bar{B} D$$

## Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \bar{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \bar{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \bar{x}y = x + y$

(15b)  $\bar{x} + xy = \bar{x} + y$

(16)  $\overline{(x + y)} = \bar{x} \cdot \bar{y}$

(17)  $\overline{(x \cdot y)} = \bar{x} + \bar{y}$

# Exemplos

## Example 1

$$\begin{aligned}z &= \overline{A + \overline{B \cdot C}} \\ &= \overline{A} \cdot \overline{\overline{B \cdot C}} \\ &= \overline{A} \cdot (\overline{\overline{B}} + \overline{\overline{C}}) \\ &= \overline{A} \cdot (B + \overline{C})\end{aligned}$$

## Example 2

$$\begin{aligned}\omega &= \overline{(A + BC) \cdot (D + EF)} \\ &= \overline{(A + BC)} + \overline{(D + EF)} \\ &= (\overline{A} \cdot \overline{BC}) + (\overline{D} \cdot \overline{EF}) \\ &= [\overline{A} \cdot (\overline{B} + \overline{C})] + [\overline{D} \cdot (\overline{E} + \overline{F})] \\ &= \overline{A}\overline{B} + \overline{A}\overline{C} + \overline{D}\overline{E} + \overline{D}\overline{F}\end{aligned}$$

### Teoremas:

(1)  $x \cdot 0 = 0$

(2)  $x \cdot 1 = x$

(4)  $x \cdot \overline{x} = 0$

(5)  $x + 0 = x$

(6)  $x + 1 = 1$

(8)  $x + \overline{x} = 1$

(14)  $x + xy = x$

(15a)  $x + \overline{x}y = x + y$

(15b)  $\overline{x} + xy = \overline{x} + y$

(16)  $\overline{(x + y)} = \overline{x} \cdot \overline{y}$

(17)  $\overline{(x \cdot y)} = \overline{x} + \overline{y}$

### Note:

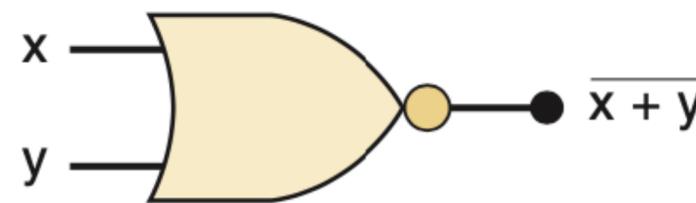
$$\overline{x + y + z} = \overline{x} \cdot \overline{y} \cdot \overline{z}$$

$$\overline{x \cdot y \cdot z} = \overline{x} + \overline{y} + \overline{z}$$

# Implicações do Teoremas de Demorgan's

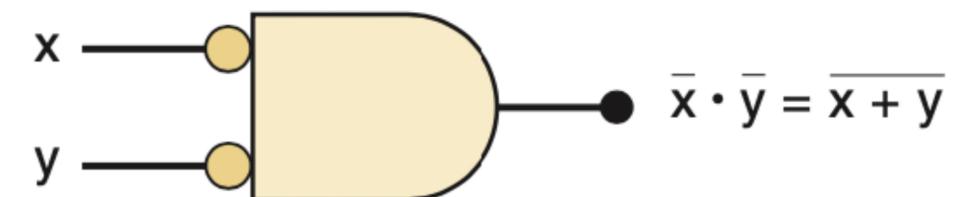
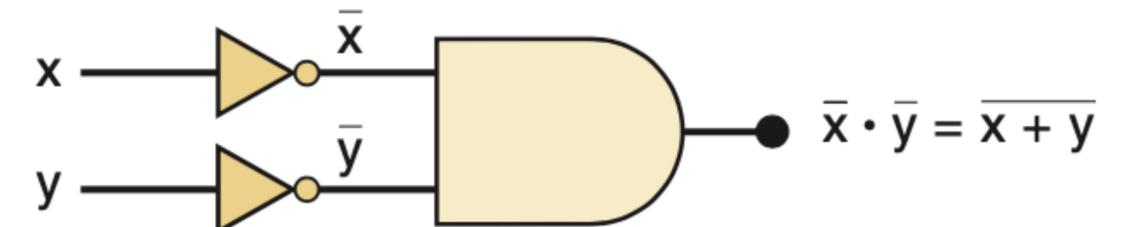
- Universalidade das portas **NOR** e **NAND**:

$$(16) \overline{(x + y)} = \bar{x} \cdot \bar{y}$$

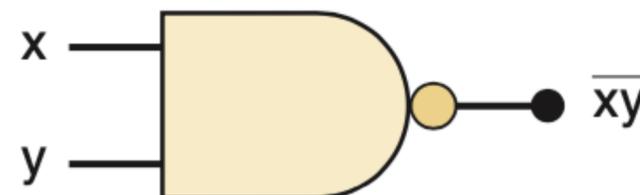


NOR

≡

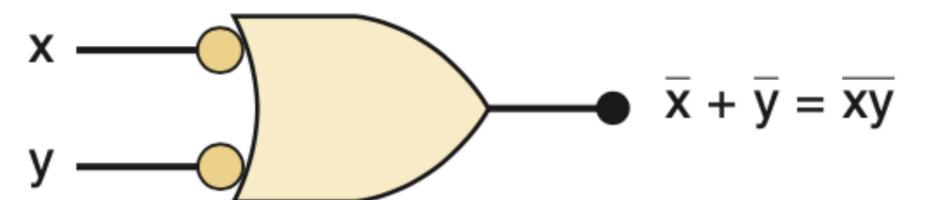
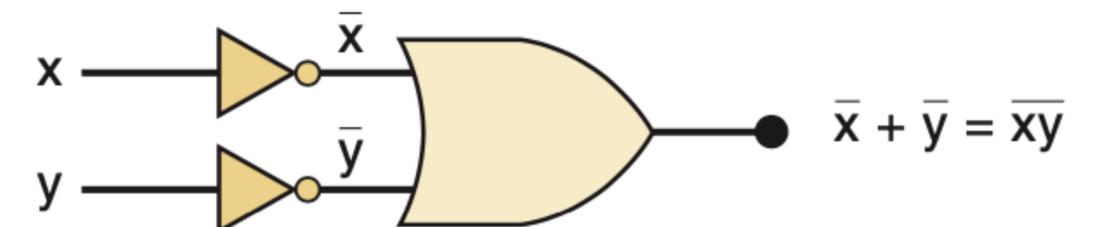


$$(17) \overline{(x \cdot y)} = \bar{x} + \bar{y}$$



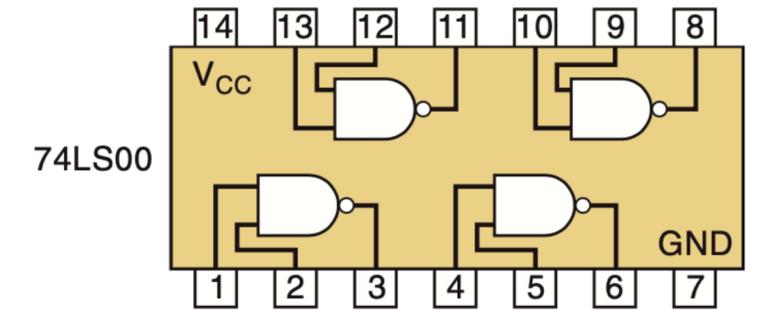
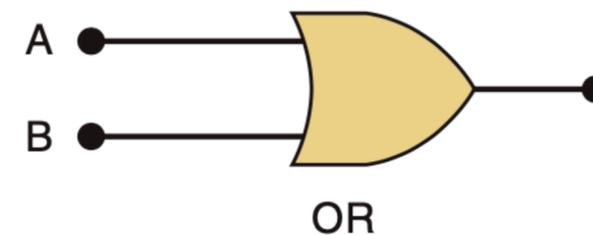
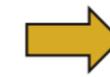
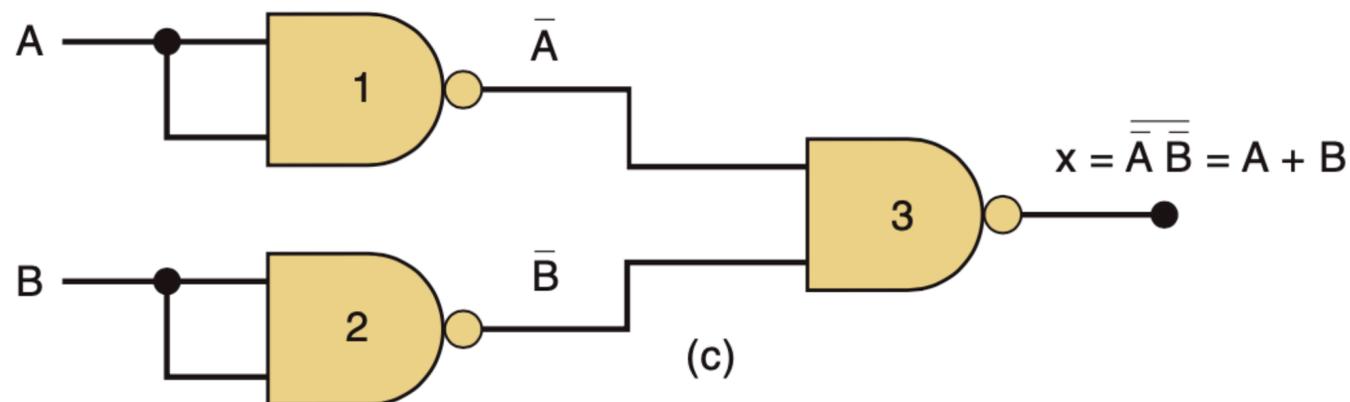
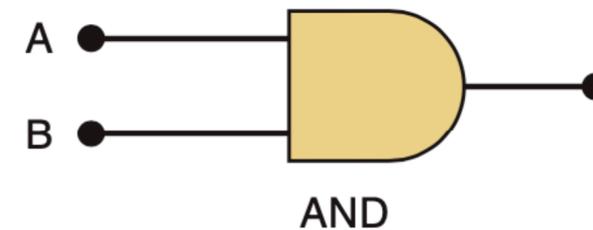
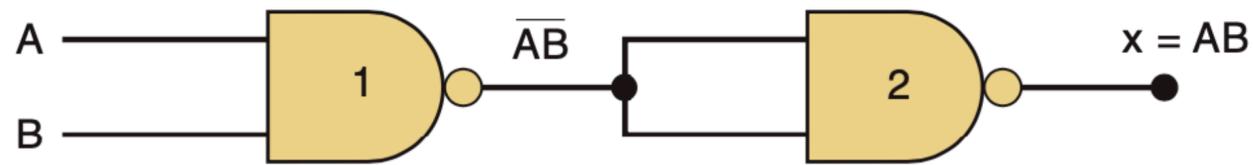
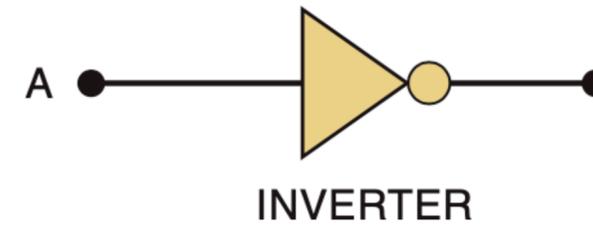
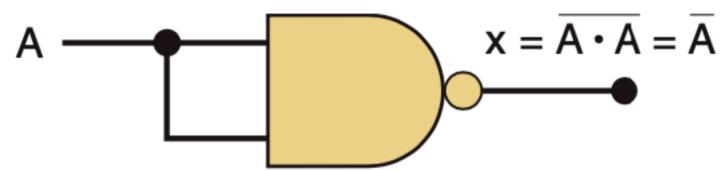
NAND

≡



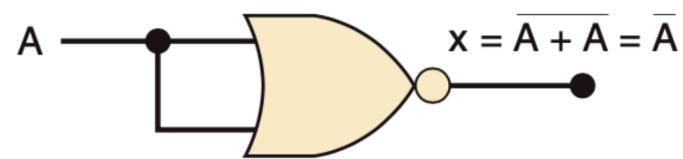
# Implicações do Teoremas de Demorgan's

- Universalidade das portas **NAND**: (16)  $(\overline{x + y}) = \bar{x} \cdot \bar{y}$

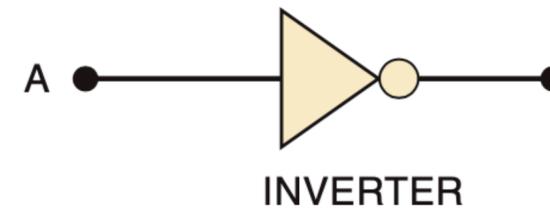
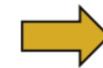


# Implicações do Teoremas de Demorgan's

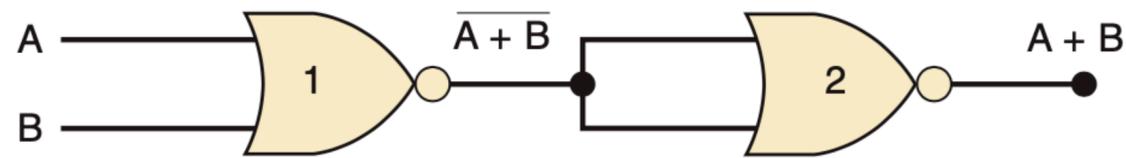
- Universalidade das portas **NOR**: (17)  $(\overline{x \cdot y}) = \bar{x} + \bar{y}$



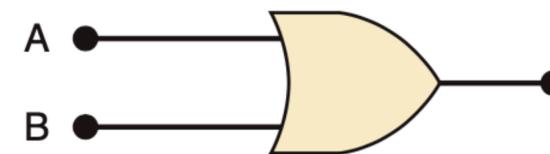
(a)



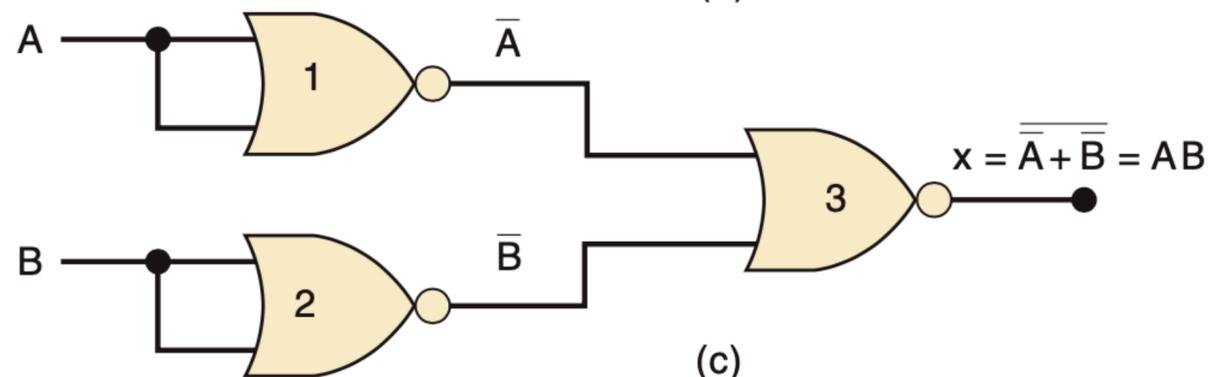
INVERTER



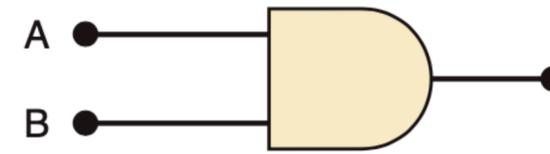
(b)



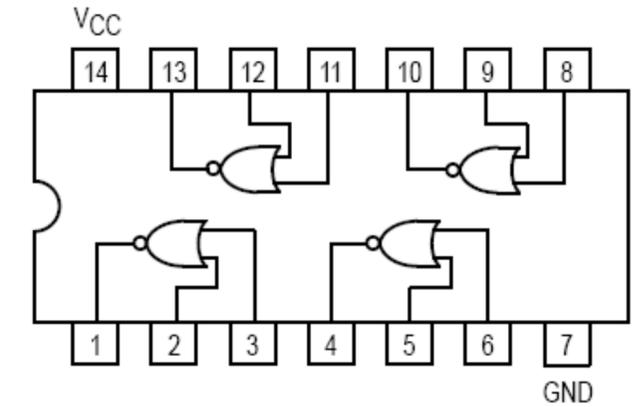
OR



(c)



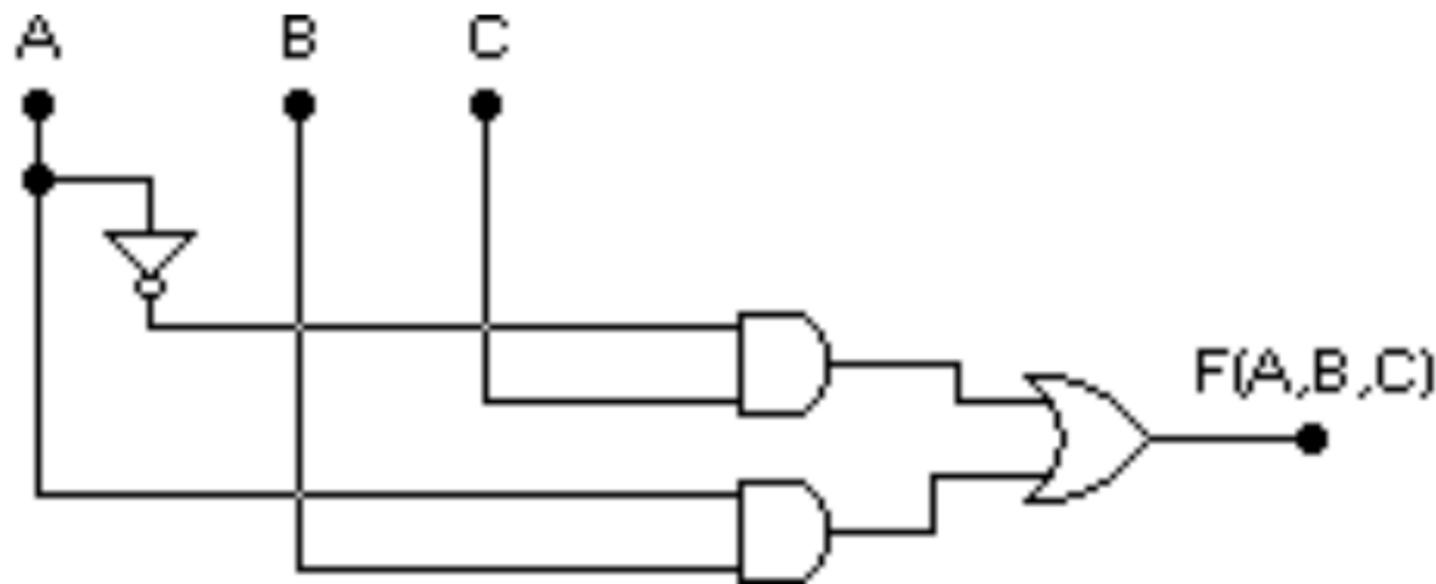
AND



74LS02

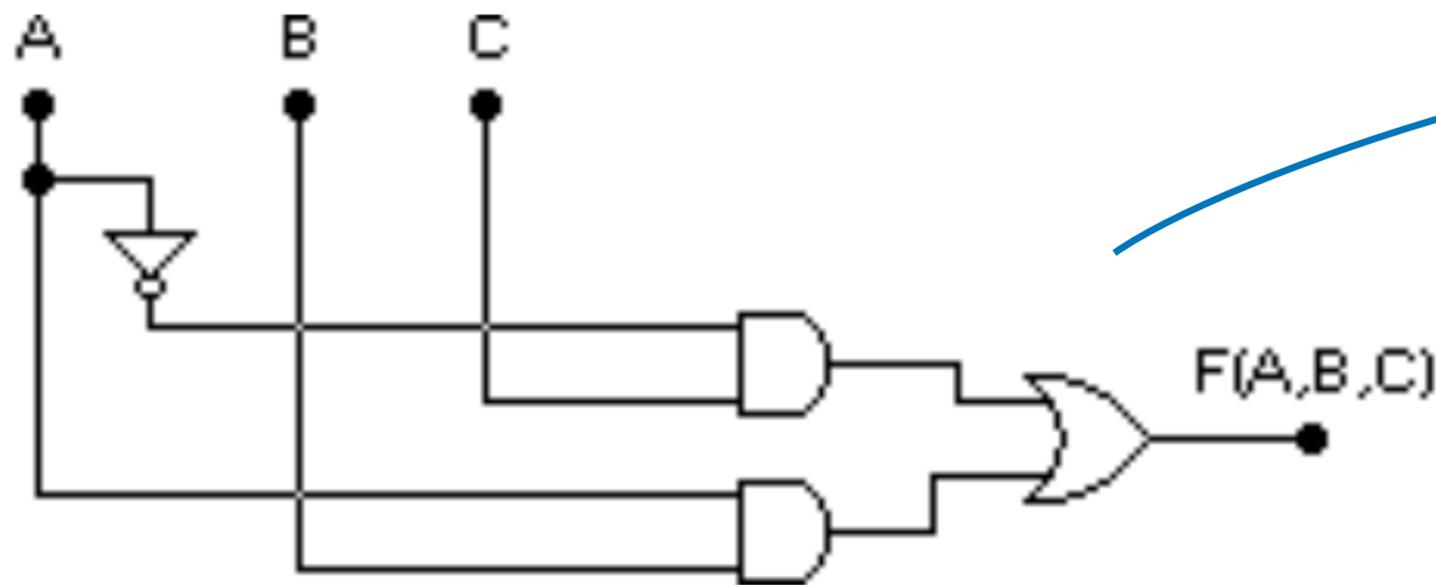
# (Custo de) Síntese de Circuitos

- Comprove usando álgebra de Boole que o circuito abaixo pode ser realizado usando-se apenas 01 (um) CI 74LS00 (4 x NAND(2)):

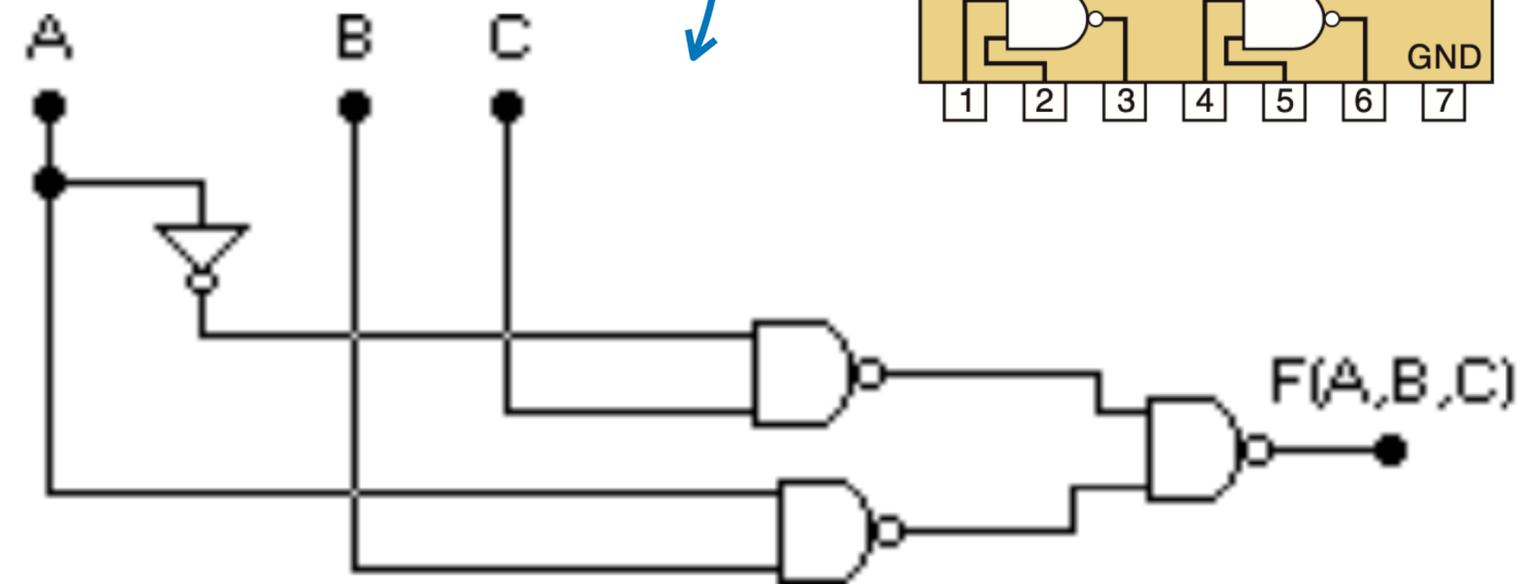
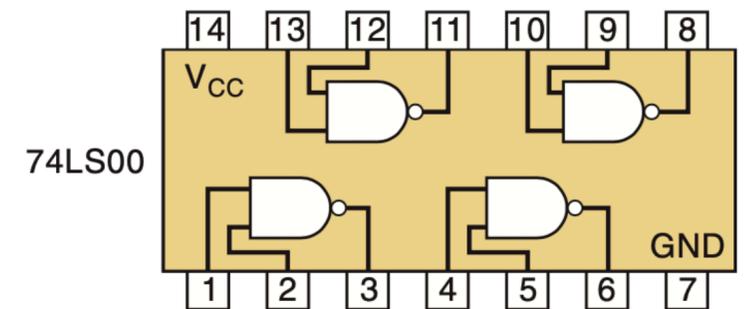


# (Custo de) Síntese de Circuitos

- Comprove usando álgebra de Boole que o circuito abaixo pode ser realizado usando-se apenas 01 (um) CI 74LS00 (4 x NAND(2)):

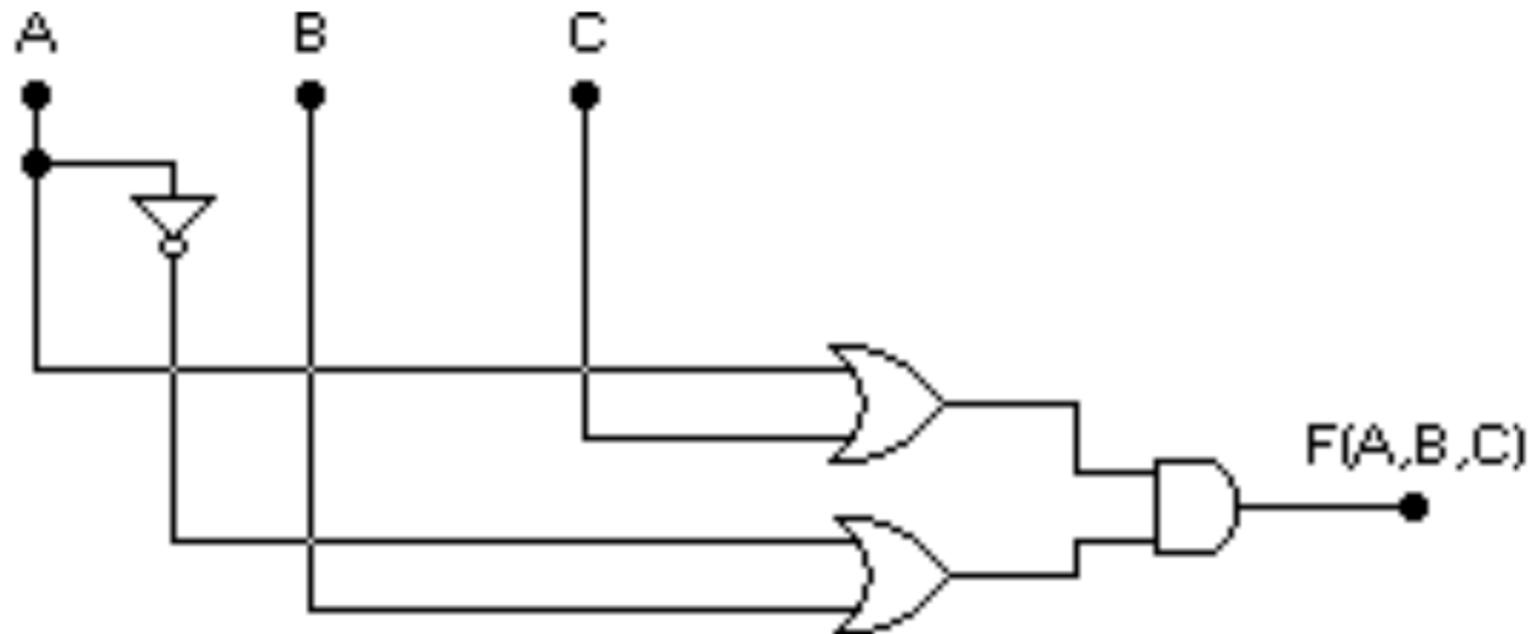


Solução:



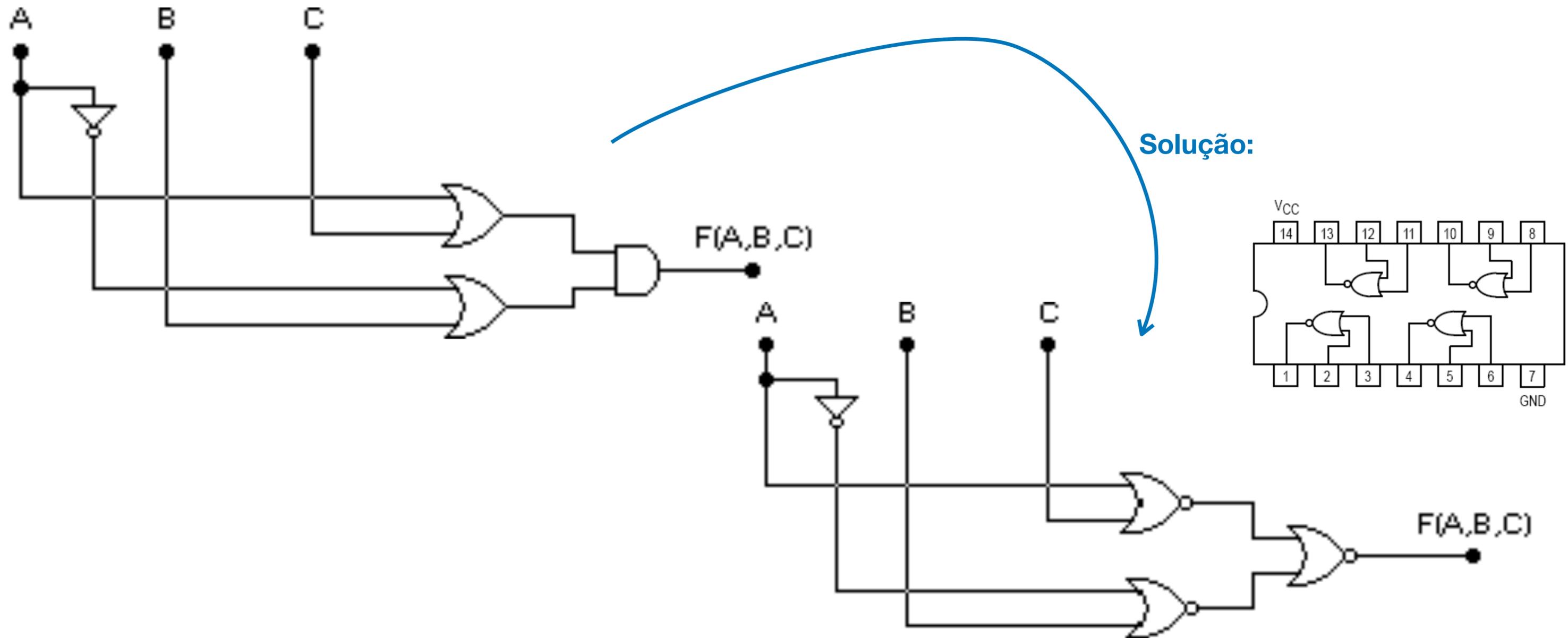
# (Custo de) Síntese de Circuitos

- Comprove usando álgebra de Boole que o circuito abaixo pode ser realizado usando-se apenas 01 (um) CI 74LS02 (4 x NOR(2)):

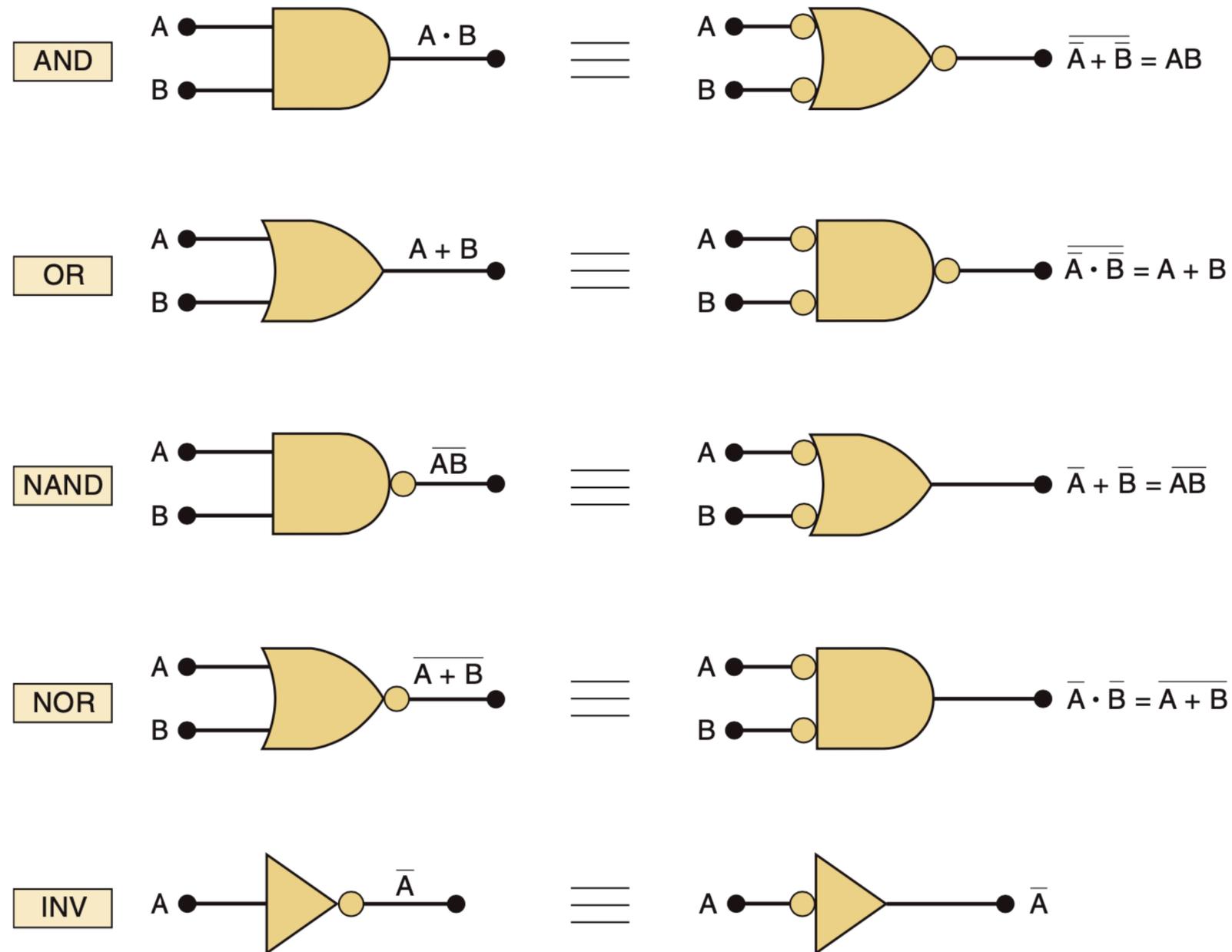


# (Custo de) Síntese de Circuitos

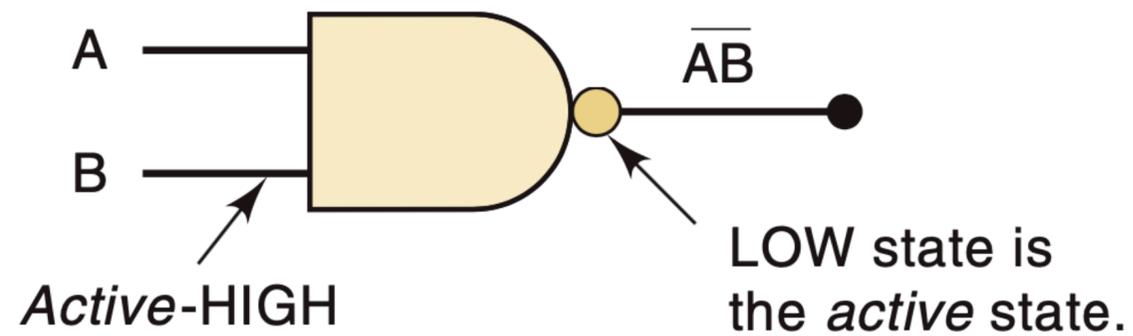
- Comprove usando álgebra de Boole que o circuito abaixo pode ser realizado usando-se apenas 01 (um) CI 74LS02 (4 x NOR(2)):



# Símbolos Alternativos

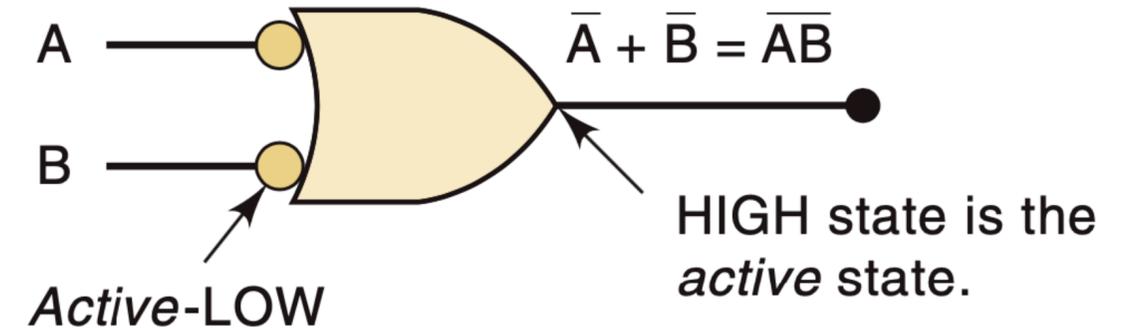


# “Ativo ALTO” x “Ativo BAIXO”



**NAND**

Note:  
MESMA Porta

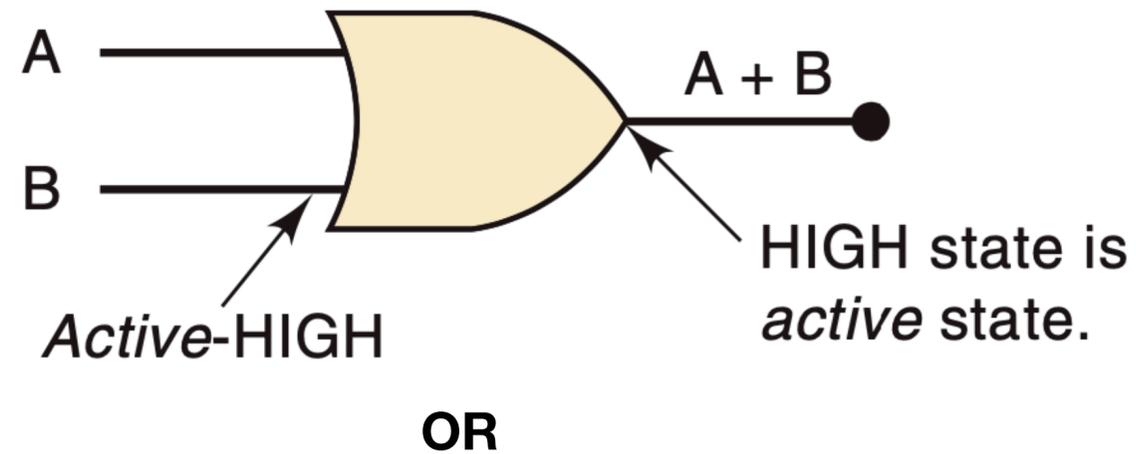


**≡ NAND**

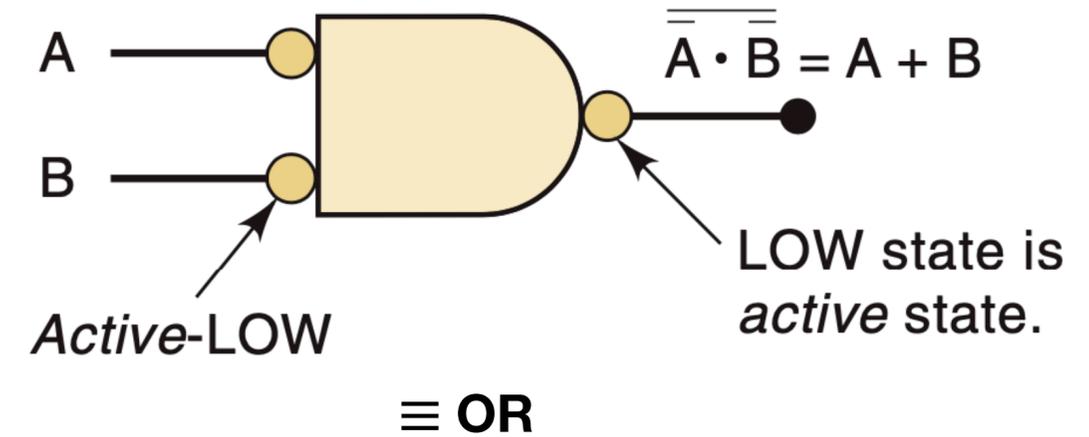
Saída vai a nível lógico BAIXO apenas quando TODAS as entradas estiverem em nível lógico ALTO.

AB	AND	NAND
0 0	0	1
0 1	0	1
1 0	0	1
1 1	1	0

# “Ativo ALTO” x “Ativo BAIXO”



Note:  
MESMA Porta

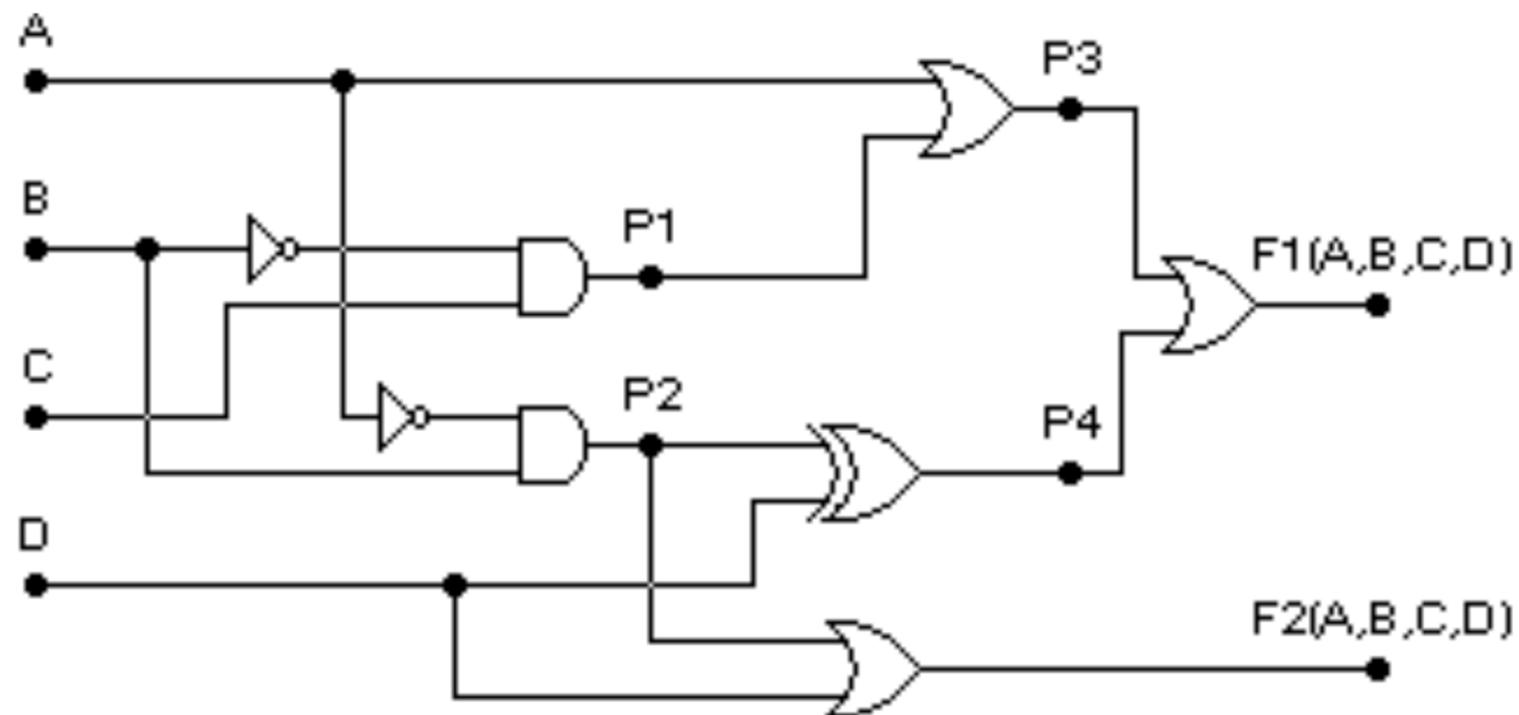


Saída vai a nível lógico ALTO apenas quando QUALQUER entradas estiver em nível lógico ALTO.

AB	OR
0 0	0
0 1	1
1 0	1
1 1	1

# Análise de Circuitos

- Deduza as expressões (e/ou tabela verdade para F1 e F2):



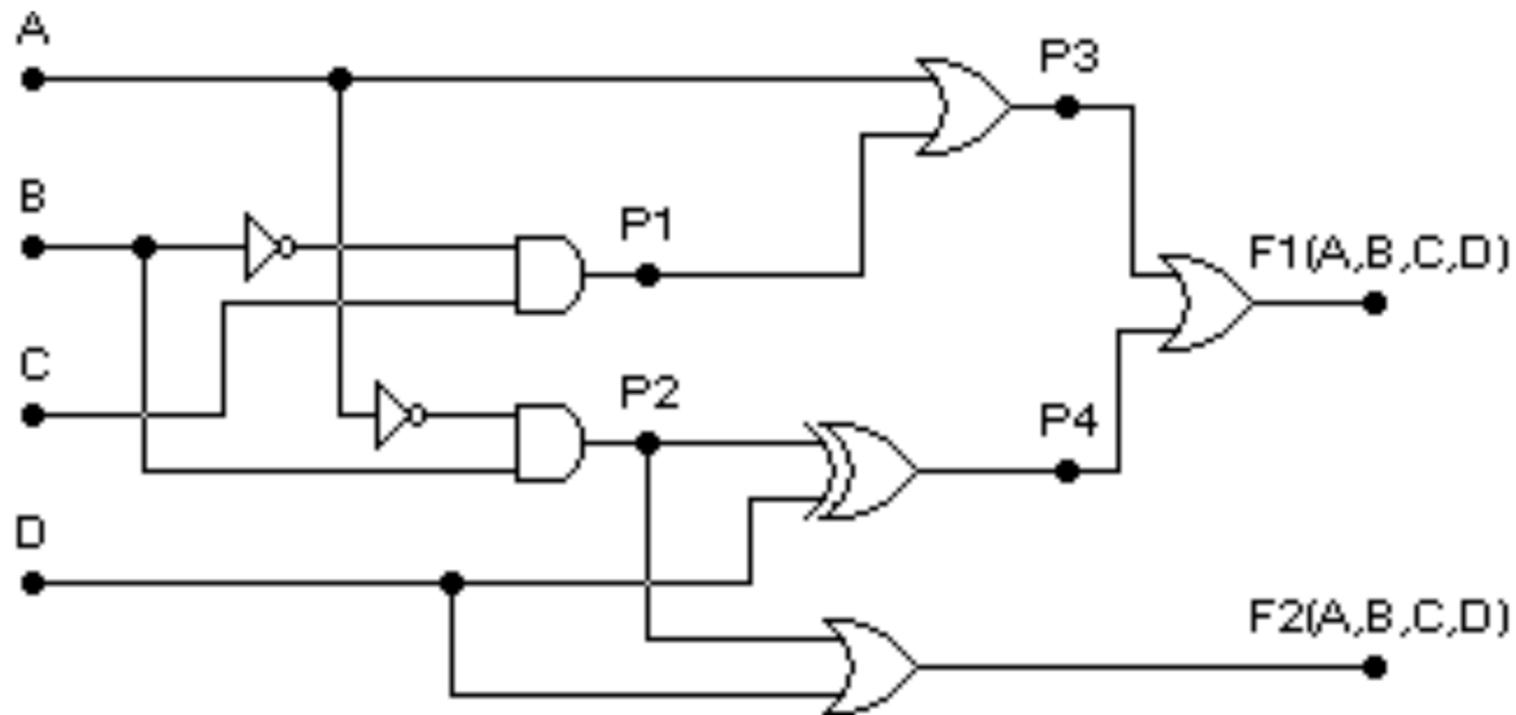
Solução:

$$F1 = A + B\bar{D} + \bar{B}C + \bar{B}D$$

$$F2 = \bar{A}B + D$$

# Análise de Circuitos

- Deduza as expressões (e/ou tabela verdade para F1 e F2):



Solução:

$$P1 = \bar{B} C$$

$$P2 = \bar{A} B$$

$$P3 = A + P1 = A + \bar{B} C$$

$$P4 = P2 \oplus D = P2 \bar{D} + \overline{P2} D$$

$$P4 = \bar{A} B \bar{D} + (\overline{\bar{A} B}) D$$

$$P4 = \bar{A} B \bar{D} + (\bar{A} + \bar{B}) D$$

$$P4 = \bar{A} B \bar{D} + AD + \bar{B} D$$

$$F1 = P3 + P4$$

$$F1 = A + \bar{B} C + \bar{A} B \bar{D} + AD + \bar{B} D$$

$$F1 = A(1 + D) + \bar{B} C + \bar{A} B \bar{D} + \bar{B} D$$

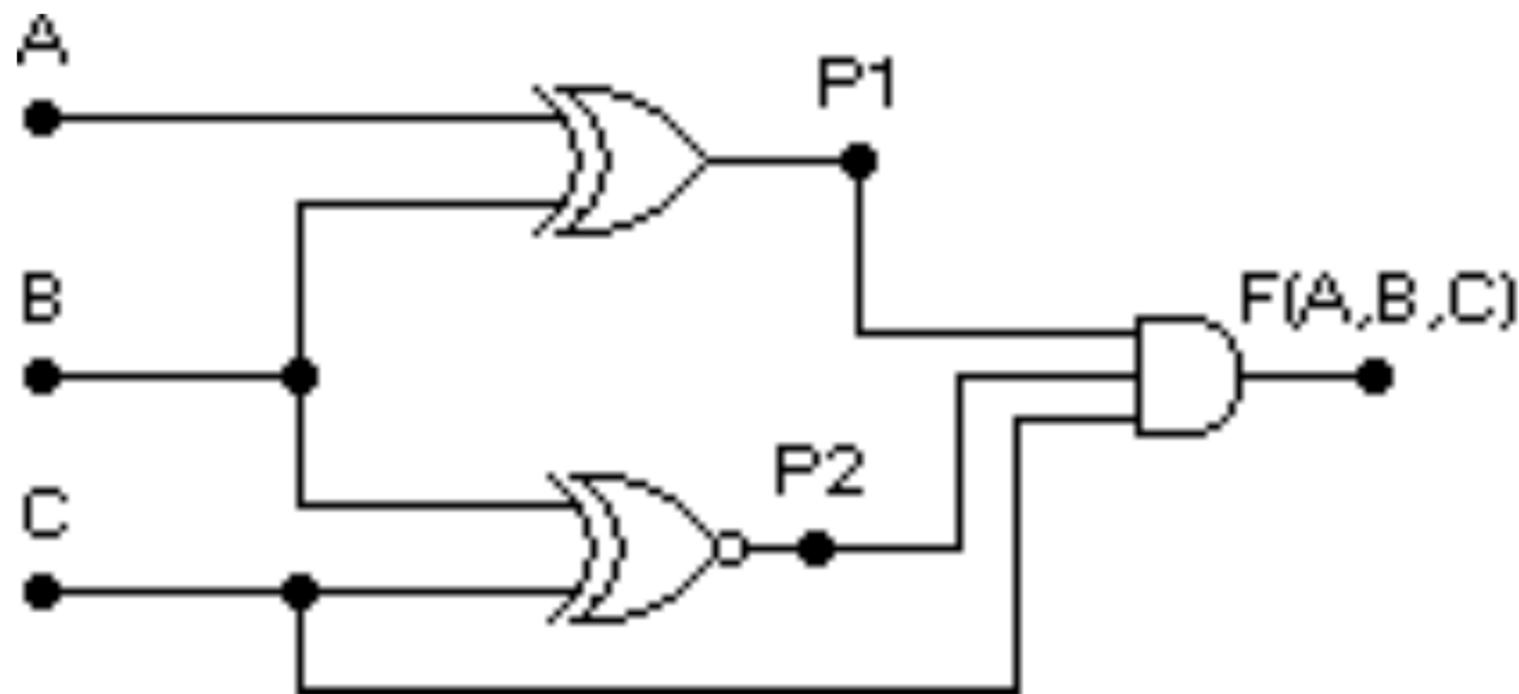
$$F1 = A + \bar{A} B \bar{D} + \bar{B} C + \bar{B} D$$

$$F1 = A + B \bar{D} + \bar{B} C + \bar{B} D$$

$$F2 = P2 + D = \bar{A} B + D$$

# Análise de Circuitos

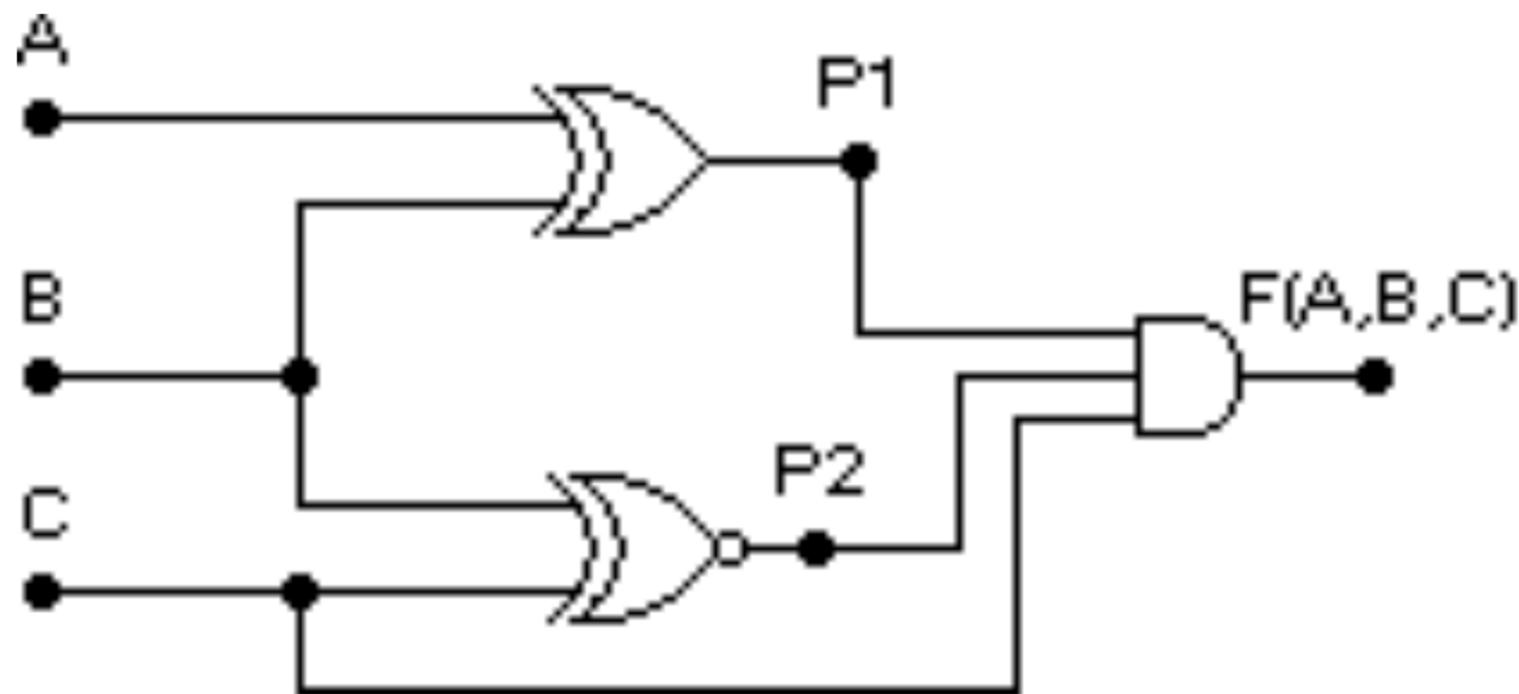
- Levante a tabela verdade de F:



Ref	A	B	C	P1	P2	F
0	0	0	0			
1	0	0	1			
2	0	1	0			
3	0	1	1			
4	1	0	0			
5	1	0	1			
6	1	1	0			
7	1	1	1			

# Análise de Circuitos

- Levante a tabela verdade de F:

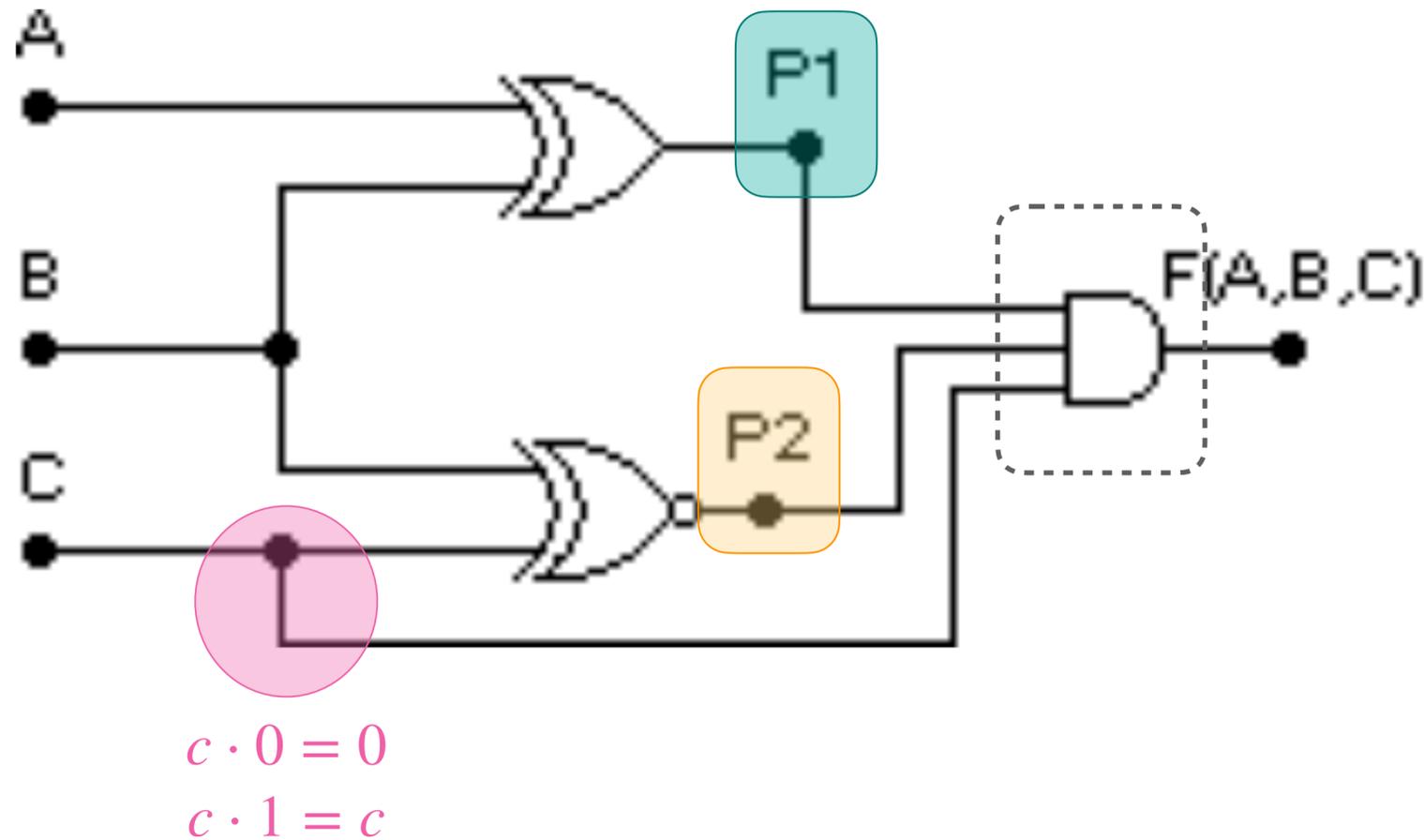


Solução:

Ref	A B C	P1	P2	F
0	0 0 0	0	1	0
1	0 0 1	0	0	0
2	0 1 0	1	0	0
3	0 1 1	1	1	1
4	1 0 0	1	1	0
5	1 0 1	1	0	0
6	1 1 0	0	0	0
7	1 1 1	0	1	0

# Análise de Circuitos

- Levante a tabela verdade de F:



Solução:

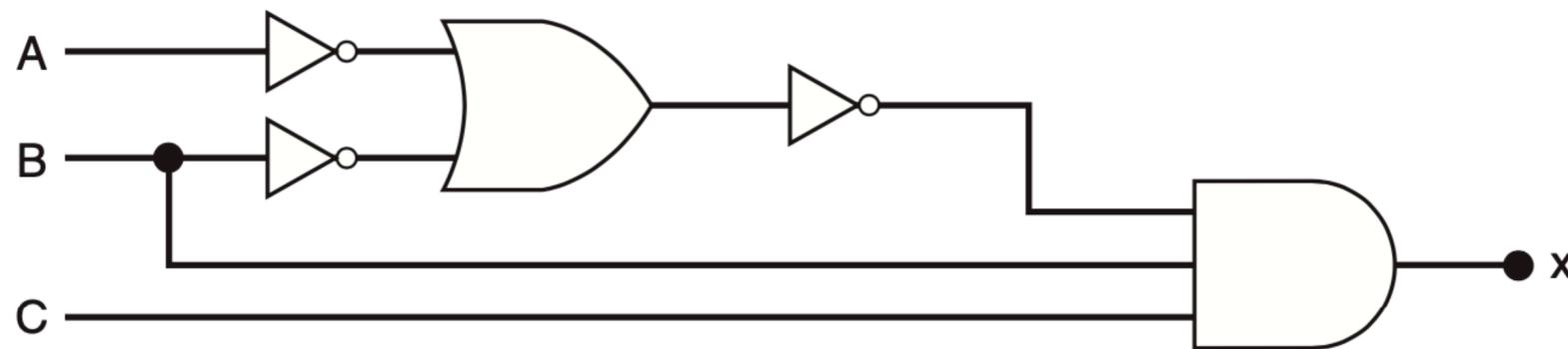
Ref	A	B	C	P1	P2	F
0	0	0	0	0	1	0
1	0	0	1	0	0	0
2	0	1	0	1	0	0
3	0	1	1	1	1	1
4	1	0	0	1	1	0
5	1	0	1	1	0	0
6	1	1	0	0	0	0
7	1	1	1	0	1	0

**P1: porta XOR: Detector de Desigualdade (A,B)**

**P2: porta NXOR: Igualdades (B,C)**

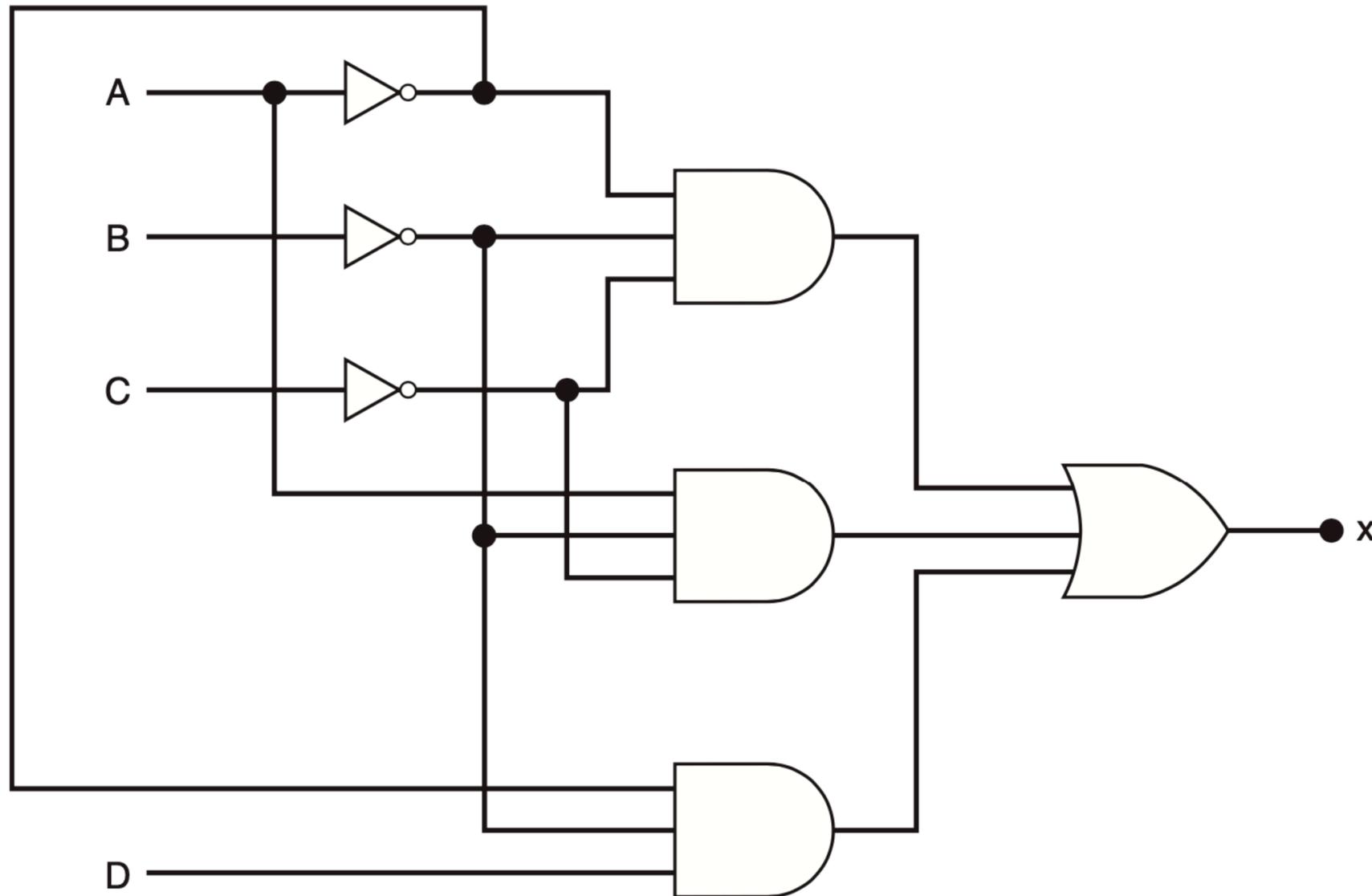
# Análise de Circuitos

- Escreva a equação para  $x$  e levante a tabela verdade do circuito abaixo



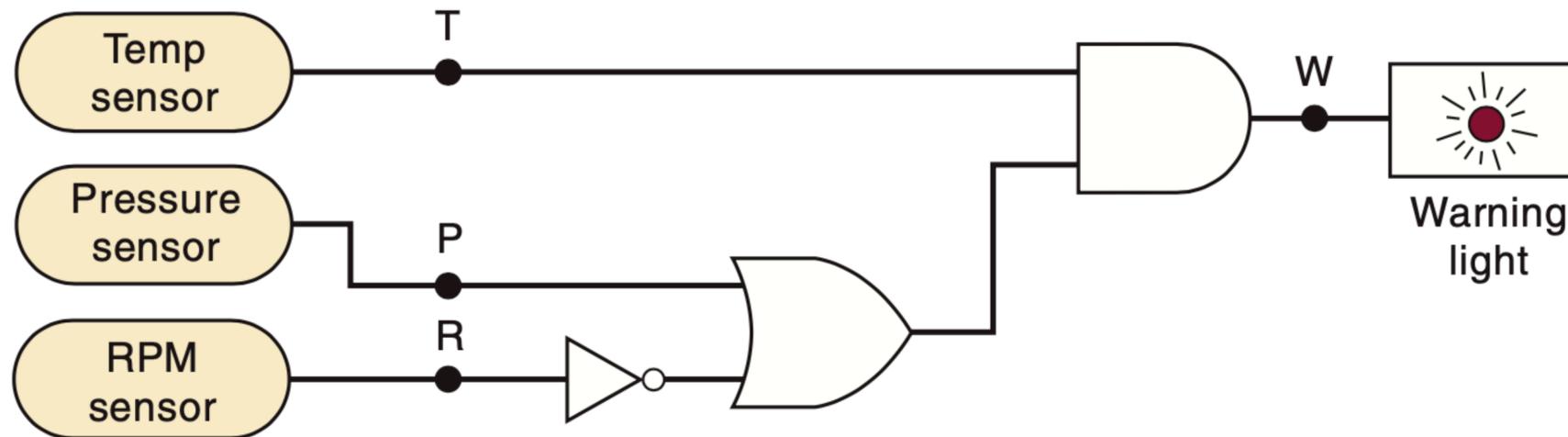
# Análise de Circuitos

- Escreva a equação para  $x$  e levante a tabela verdade do circuito abaixo



# Análise de Circuitos

- Identifique sob que condições a lâmpada de advertência mostrada no circuito abaixo é ativada?



- Note que este circuito faz parte do sistema de monitoramento do motor de um avião, usando sensores que operam da seguinte forma:

Sensor R = 0 apenas quando Velocidade < 4800 RPM

Sensor P = 0 apenas quando Pressão < 220 psi

Sensor T = 0 apenas quando Temperatura < 200°F