Mapas de Karnaugh

Circuitos Digitais I

Prof. Fernando Passold

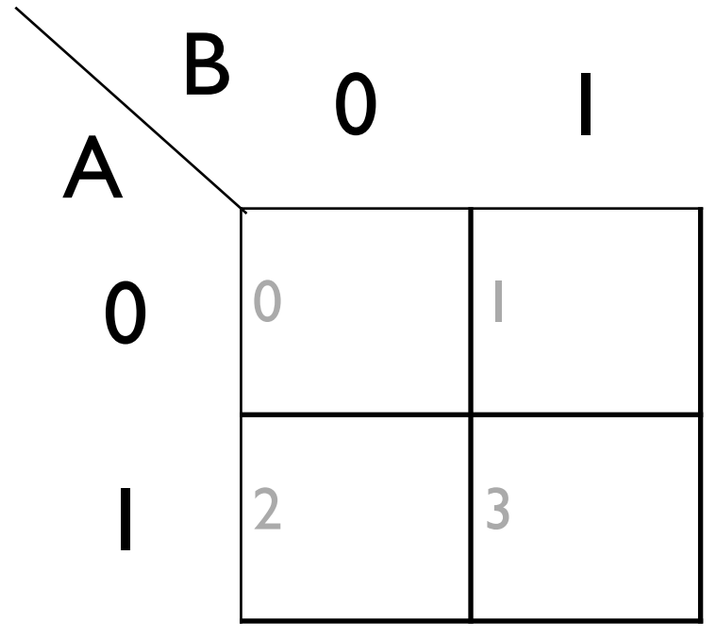
Introdução

- Origem:
  - Desenvolvido em 1953 por Maurice Karnaugh, um engenheiro de telecomunicações da Bell Labs
- Objetivo:
  - Reduzir (simplificar) expressões lógicas.

Lógica do Mapa

- Representar tabela verdade num outro formato (gráfico) “matricial”.
- Exemplo: Seja uma tabela verdade para 2 variáveis de entrada:

Ref	A	B	Saída
0	0	0	
1	0	1	
2	1	0	
3	1	1	



Lógica do Mapa

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Lógica do Mapa

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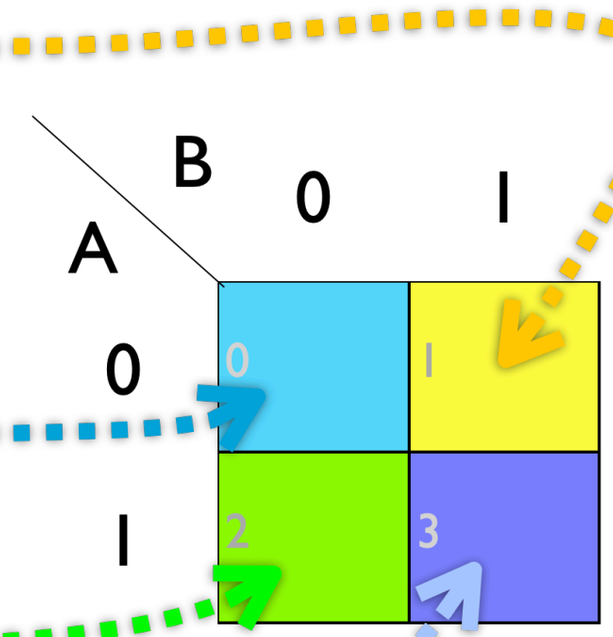


Lógica do Mapa

- Representar tabela verd num outro formato (gráfico) “matricial”.
- Exemplo: Seja uma de para 2 variáveis de entrada:

Que Falta ?

Ref	A	B	Saída
0	0	0	
1	0	1	
2	1		
3	1		

A \ B	0	1
0	0	1
1	2	3

Completar a tabela verdade e o mapa!

Uso do Mapa

- Exemplo:
  Seja a função: $F = \overline{A}\,B + A\,B$

1. Completando a tabela...

Ref	A	B	Saída
0	0	0	
1	0	1	
2	1	0	
3	1	1	

Uso do Mapa

- Exemplo:
  Seja a função: $F = \overline{A}\,B + A\,B$

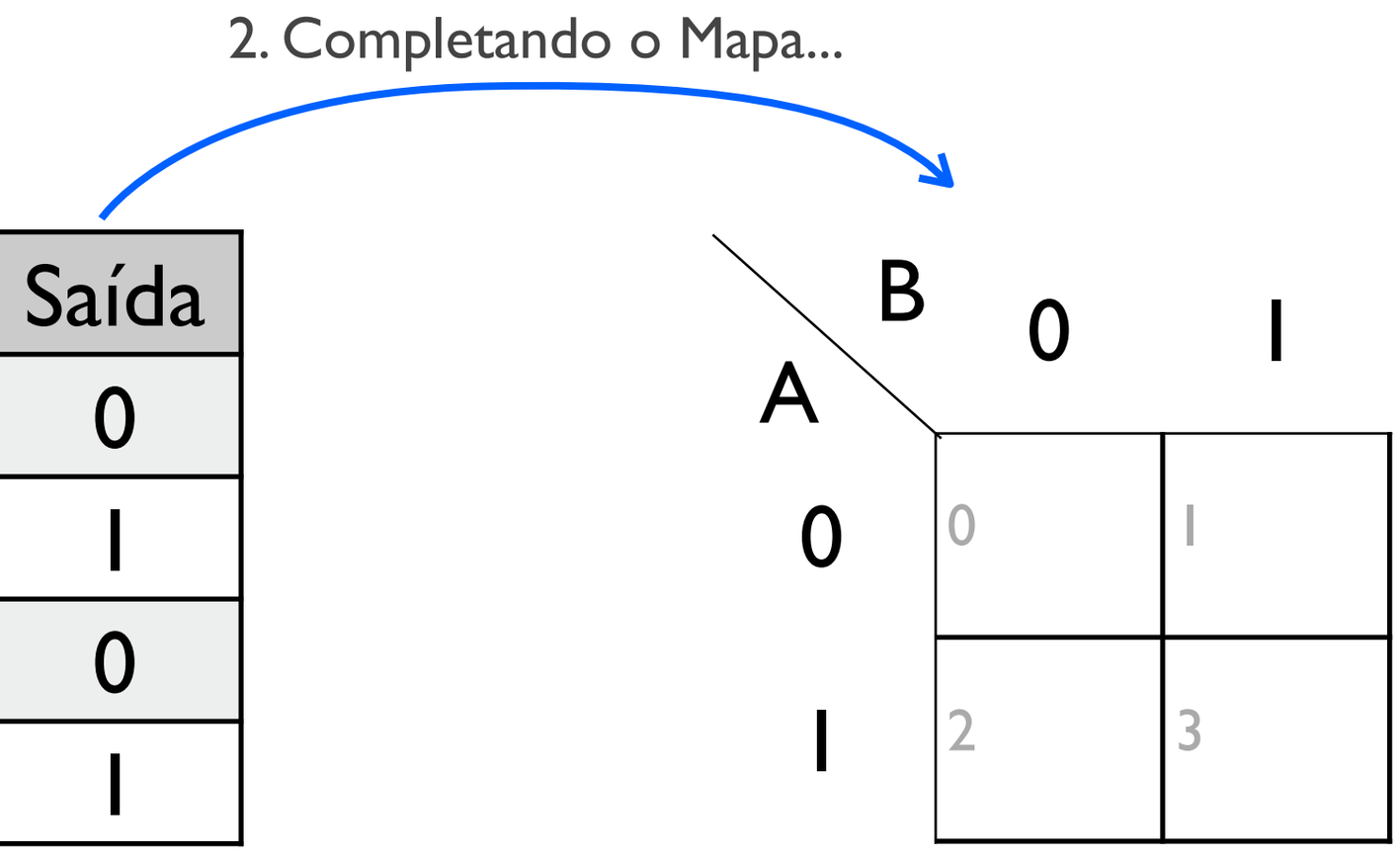
1. Completando a tabela...

Ref	A	B	Saída
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

Uso do Mapa

- Exemplo:
  Seja a função: $F = \overline{A}\,B + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1



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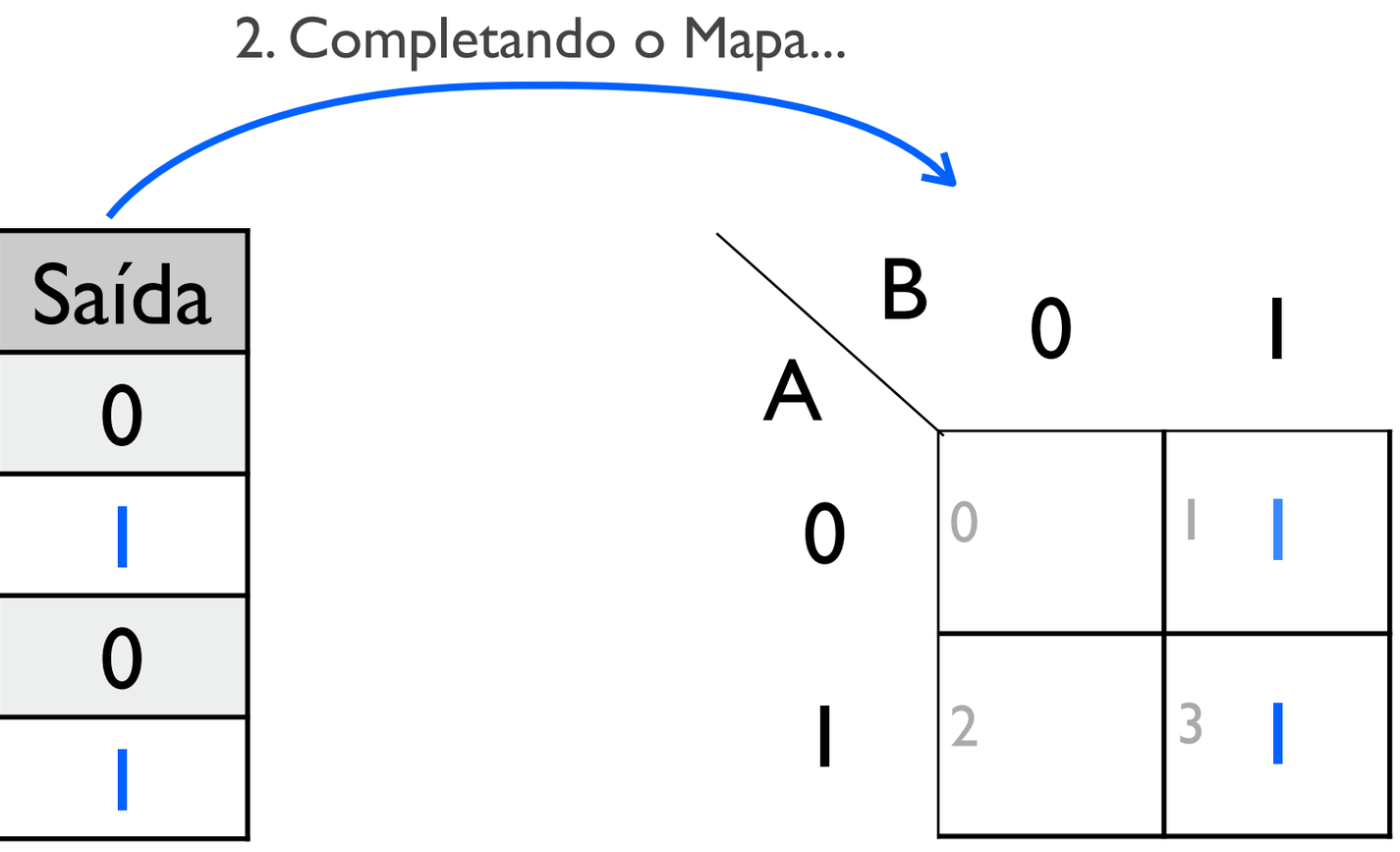
  Seja a função: $F = \overline{A}\,B + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1



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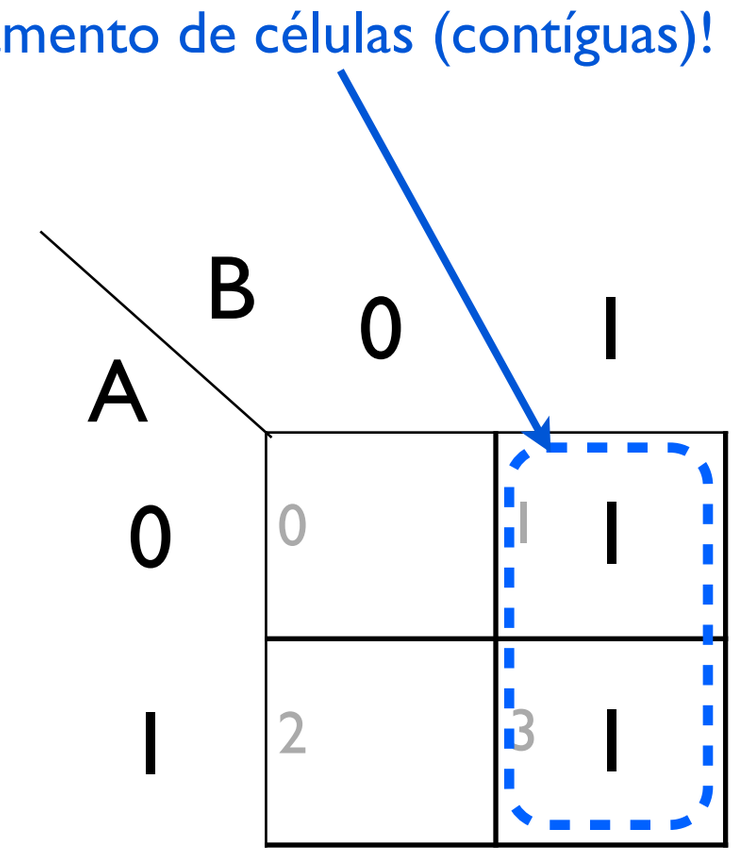
1	0	1	1
2	1	0	0
3	1	1	1



Uso do Mapa

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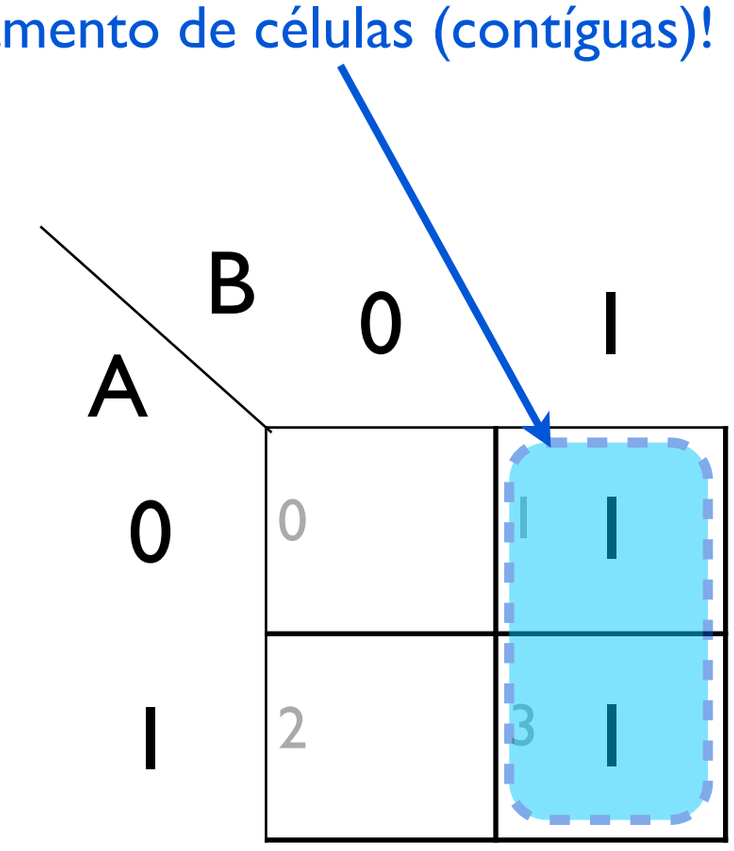
Ref	A	B	Saída
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1



Uso do Mapa

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Ref	A	B	Saída
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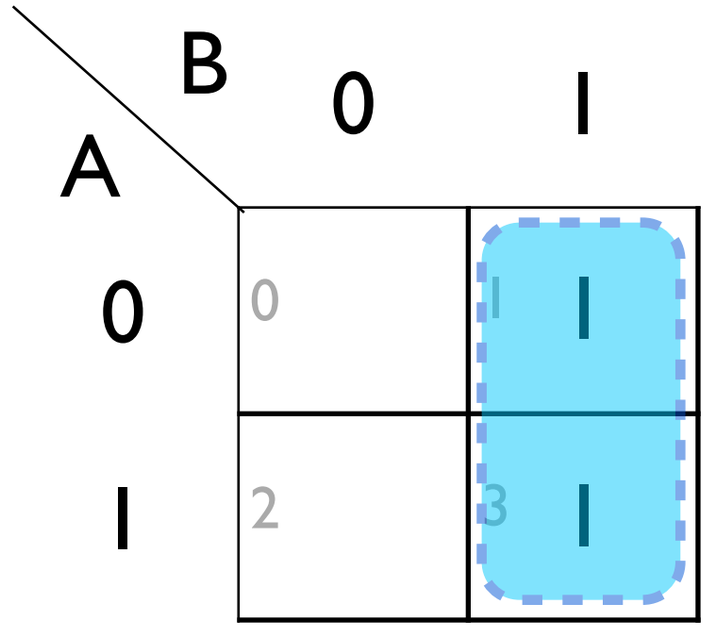
1	0	1	1
2	1	0	0
3	1	1	1



Uso do Mapa

- Exemplo:
  Seja a função: $F = \overline{A}\,B + A\,B$

Ref	A	B	Saída
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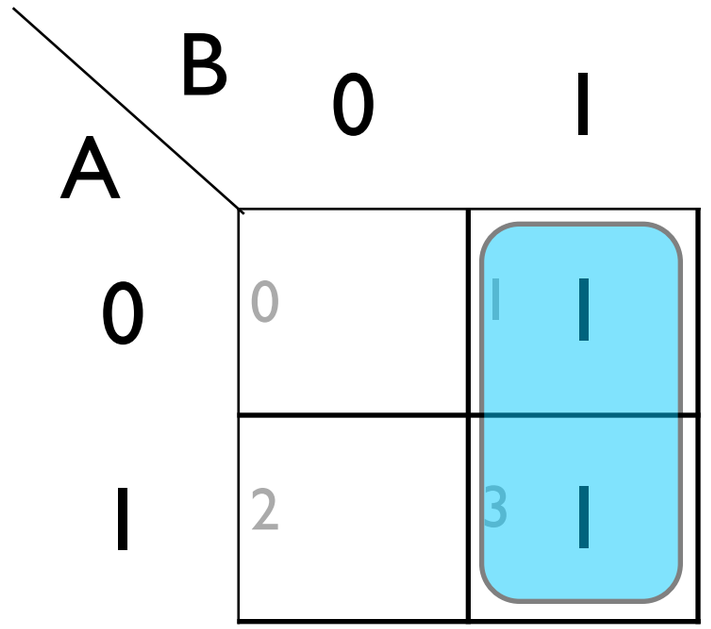
1	0	1	1
2	1	0	0
3	1	1	1

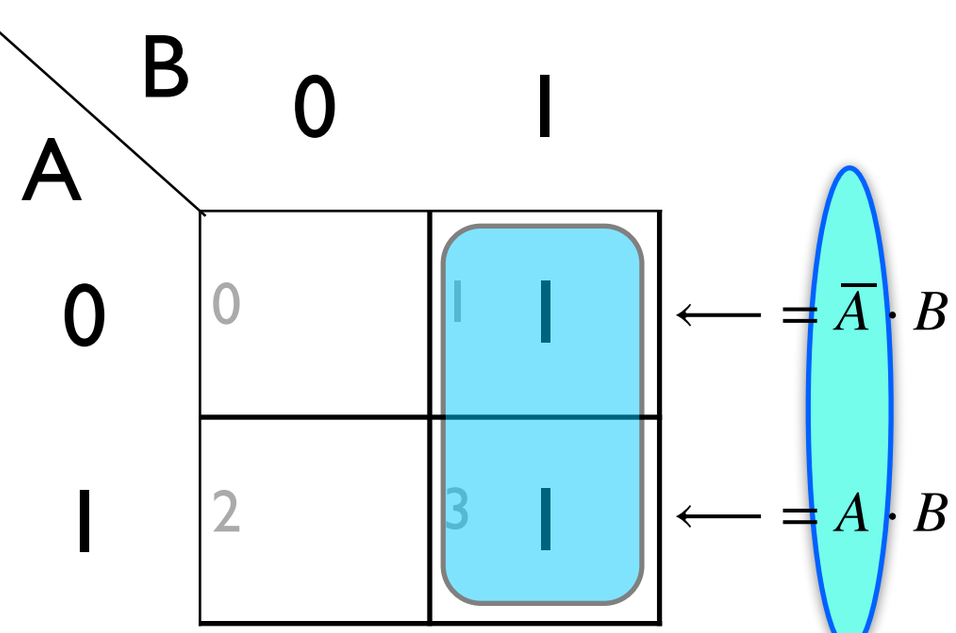


$$F = \overline{A}B + AB$$
$$F = B\left(\overline{A} + A\right)$$
$$F = B$$

Uso do Mapa

- Exemplo:
  Seja a função: $F = \overline{A}\,B + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1



$$F = \overline{A}B + AB$$
$$F = B\left(\overline{A} + A\right)$$
$$F = B$$

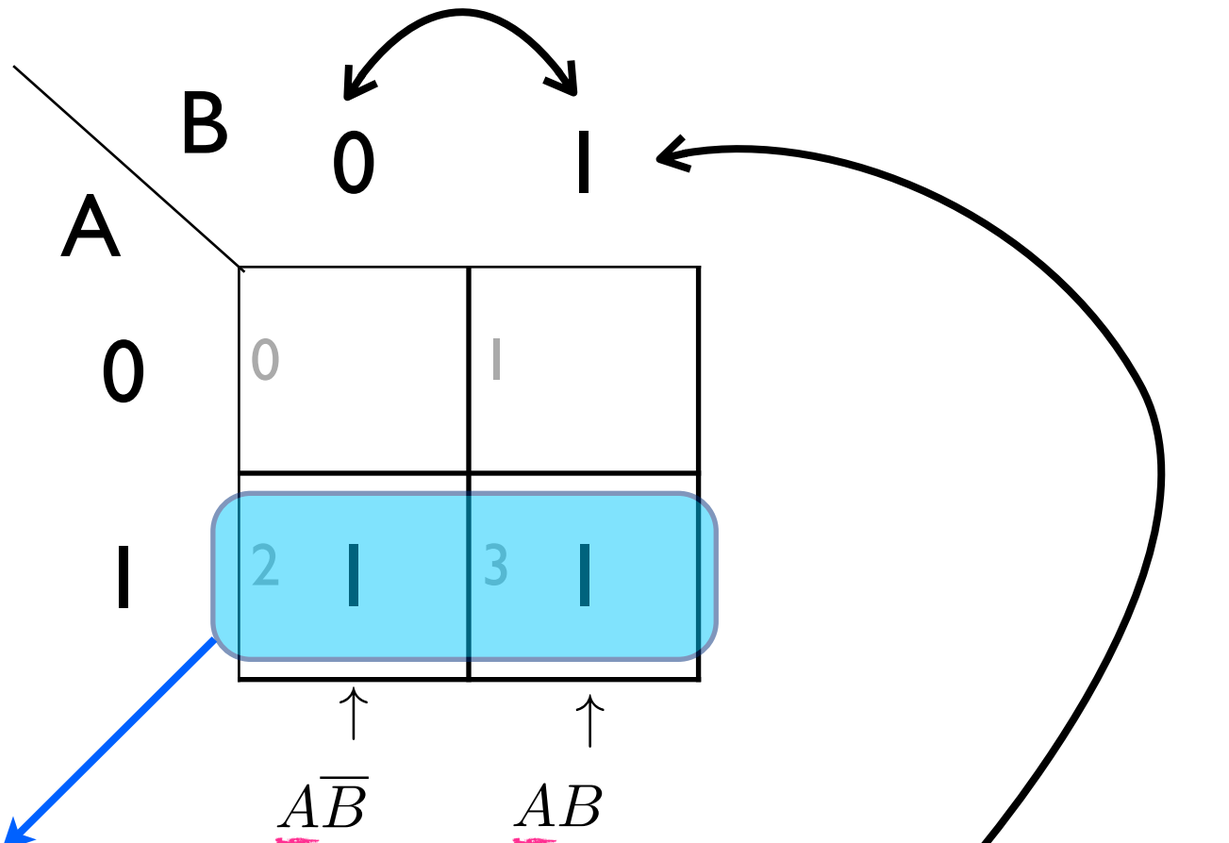
Variável que mudou de nível lógico:
Variável eliminada (simplificada)

"sobra"

Uso do Mapa

- Exemplo 2:
  Seja a função: $F = A\,\overline{B} + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1



$$F = A\,\overline{B} + A\,B$$
$$F = A\,(\overline{B} + B)$$
$$F = A$$

Uso do Mapa ☞ Outra forma:

Note mudança na ordem entre A e B!

- Exemplo$_2$:
  Seja a função: $F = A\,\overline{B} + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1

B \ A	0	1
0	0	2 1
1	1	3 1

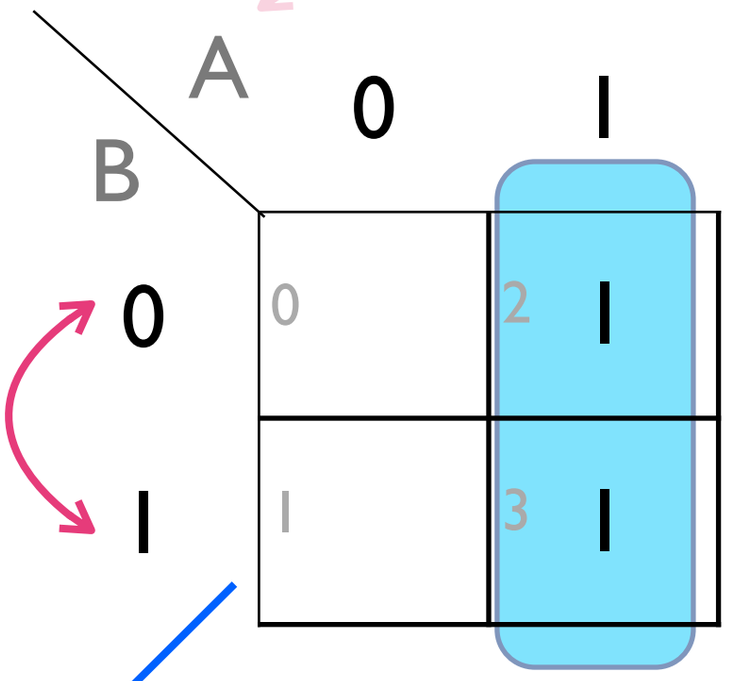
A \ B	0	1
0	0	1
1	2 1	3 1

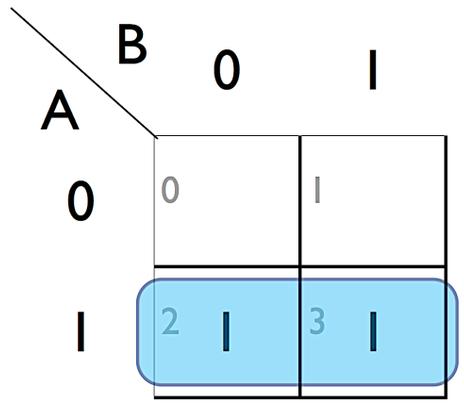
Uso do Mapa ☞ Outra forma:

Note mudança no ordem entre A e B!

- Exemplo_2:
  Seja a função: $F = A\,\overline{B} + A\,B$

Ref	A	B	Saída
0	0	0	0
1	0	1	0
2	1	0	1
3	1	1	1



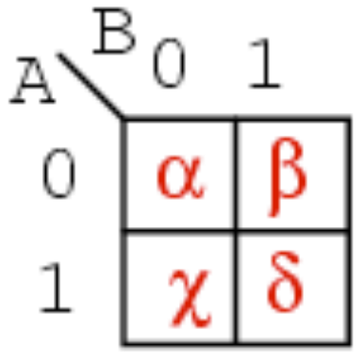


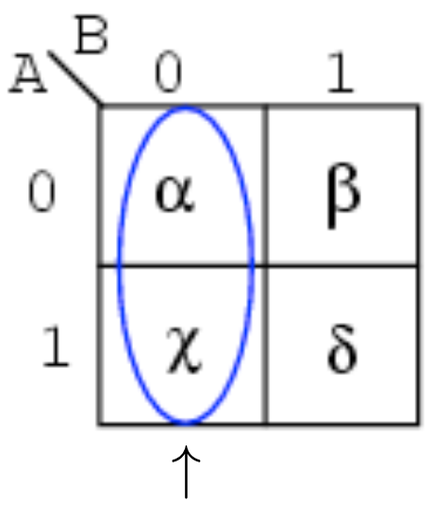
$$F = A\,\overline{B} + A\,B$$
$$F = A\,(\overline{B} + B)$$
$$F = A$$

Variável que mudou de nível:
Variável eliminada (simplificada)

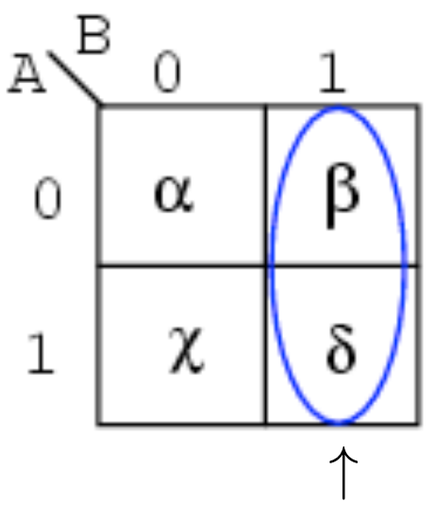
"sobra"

A	B	Output
0	0	α
0	1	β
1	0	χ
1	1	δ

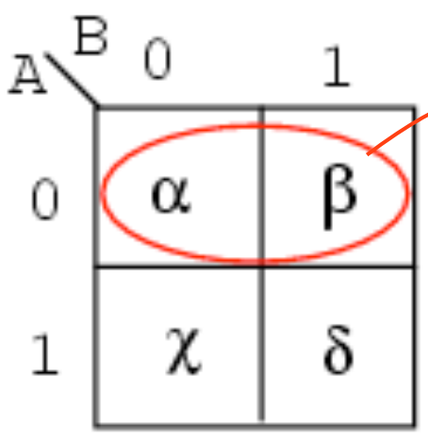




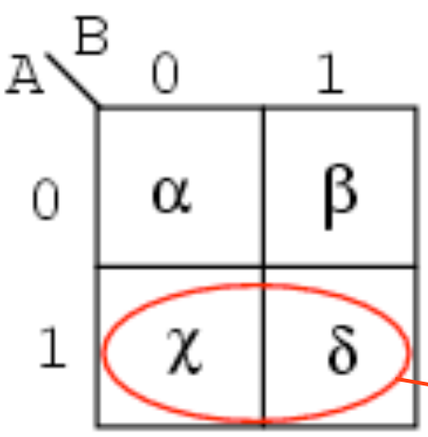
$$= \overline{A}\,\overline{B} + A\,\overline{B}$$
$$= \overline{B}(\underbrace{\overline{A} + A}_{=1})$$
$$= \overline{B}$$



$$= \overline{A}\,B + A\,B$$
$$= B(\underbrace{\overline{A} + A}_{=1})$$
$$= B$$



$$= \overline{A}\,\overline{B} + \overline{A}\,B$$
$$= \overline{A}(\underbrace{\overline{B} + B}_{=1})$$
$$= \overline{A}$$



$$= A\,\overline{B} + A\,B$$
$$= A(\underbrace{\overline{B} + B}_{=1})$$
$$= A$$

Outros Mapas para 2 variáveis

Mapa K para 3 variáveis

Ref	ABC	Y
0	000	
1	001	
2	010	
3	011	
4	100	
5	101	
6	110	
7	111	

- Mapa - Opção 1 e 2:

$A\backslash BC$

	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$AB\backslash C$

	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

Mapa K para 3 variáveis

Ref	ABC	Y
0	000	m_0
1	001	m_1
2	010	m_2
3	011	m_3
4	100	m_4
5	101	m_5
6	110	m_6
7	111	m_7

- C

Repare na ordem das células

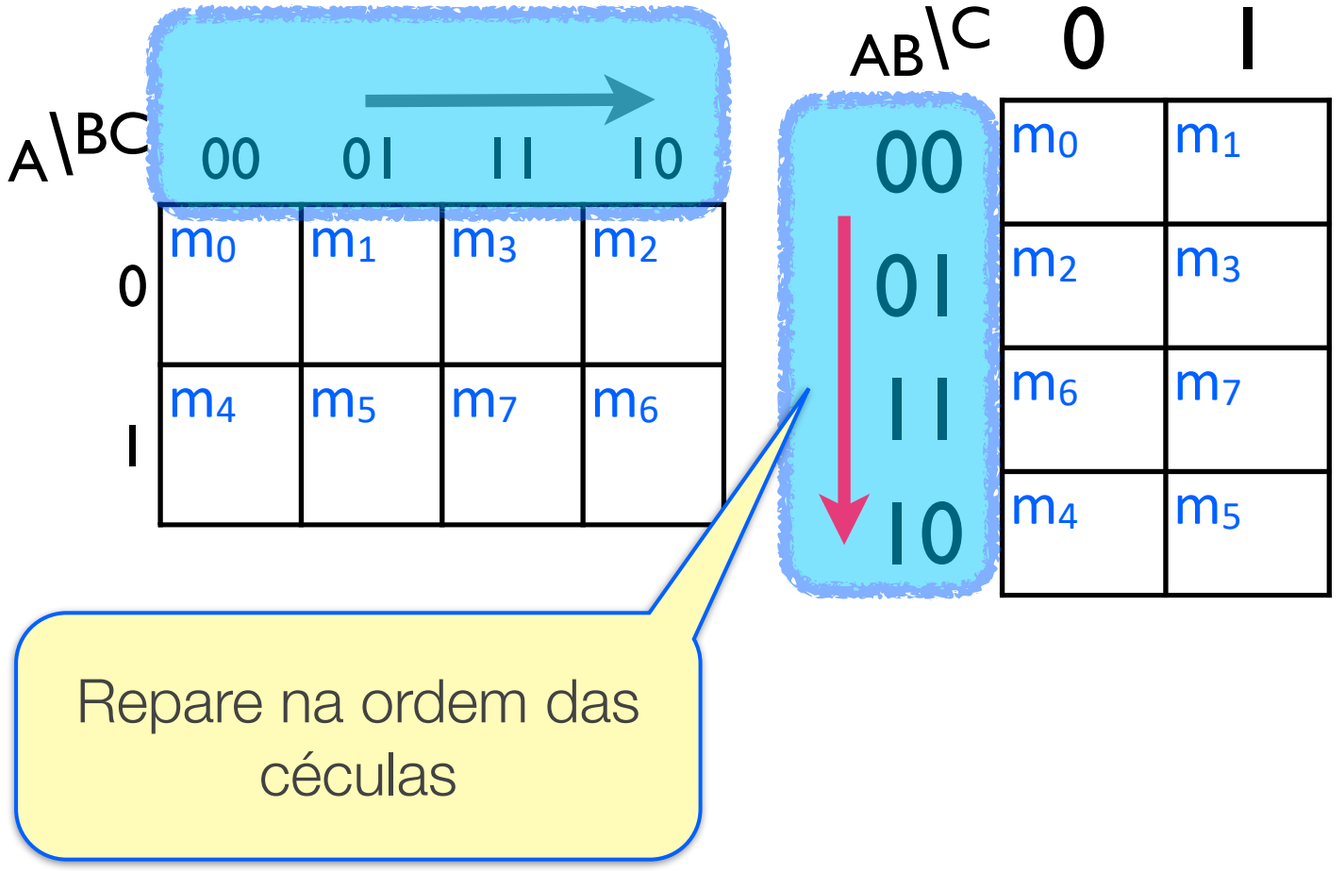
A\BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

AB\C	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

Mapa K para 3 variáveis

Ref	ABC	Y
0	000	m_0
1	001	m_1
2	010	m_2
3	011	m_3
4	100	m_4
5	101	m_5
6	110	m_6
7	111	m_7

- Ordem das células:



Mapa K para 3 variáveis

Ref	ABC	Y
0	000	m_0
1	001	m_1
2	010	m_2
3	011	m_3
4	100	m_4
5	101	m_5
6	110	m_6
7	111	m_7

- Ordem das células:

A\BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

AB\C	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

A ordem segue o código Gray (apenas 1 bit varia de estado entre células!)

Mapa K para 3 variáveis

Ref	ABC	Y
0	000	
1	001	
2	010	
3	011	
4	100	
5	101	
6	110	
7	111	

A\BC	00	01	11	10
0				
1				

AB\C	0	1
00		
01		
11		
10		

Mapa K para 3 variáveis

Ref	ABC	Y
0	000	m_0
1	001	m_1
2	010	m_2
3	011	m_3
4	100	m_4
5	101	m_5
6	110	m_6
7	111	m_7

Opção “a”:

A\BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Opção “b”:

AB\C	0	1
00	m_0	m_1
01	m_2	m_3
11	m_6	m_7
10	m_4	m_5

Exemplo_1:

Ref	ABC	Y	
0	000	1	$= \overline{A}\,\overline{B}\,\overline{C}$
1	001	1	$= \overline{A}\,\overline{B}\,C$
2	010	0	
3	011	0	
4	100	1	$= A\,\overline{B}\,\overline{C}$
5	101	0	
6	110	1	$= A\,B\,\overline{C}$
7	111	0	

- Sem Mapa:

Soma de Produtos: Minitermos

$$Y = \sum_{m} \{0, 1, 4, 6\}$$

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

$$Y = \overline{A}\,\overline{B}\,(\overline{C} + C) + A\,\overline{C}\,(\overline{B} + B)$$

$$Y = \overline{A}\,\overline{B} + A\,\overline{C}$$

Ref	ABC	Y	
0	000	1	$= \overline{A}\,\overline{B}\,\overline{C}$
1	001	1	$= \overline{A}\,\overline{B}\,C$
2	010	0	
3	011	0	
4	100	1	$= A\,\overline{B}\,\overline{C}$
5	101	0	
6	110	1	$= A\,B\,\overline{C}$
7	111	0	

Sem Mapa:

$$Y = \sum_{m} \{0, 1, 4, 6\}$$

$$Y = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$$

$$Y = \overline{A}\,\overline{B}\,(\overline{C} + C) + A\,\overline{C}\,(\overline{B} + B)$$

$$Y = \overline{A}\,\overline{B} + A\,\overline{C}$$

Com Mapa:

$Y = \overline{A}\,\overline{B} + A\,\overline{C}$

AB\C	0	1	
00	1	1	$\leftarrow \overline{A}\,\overline{B}$
01			
11	1		
10	1		

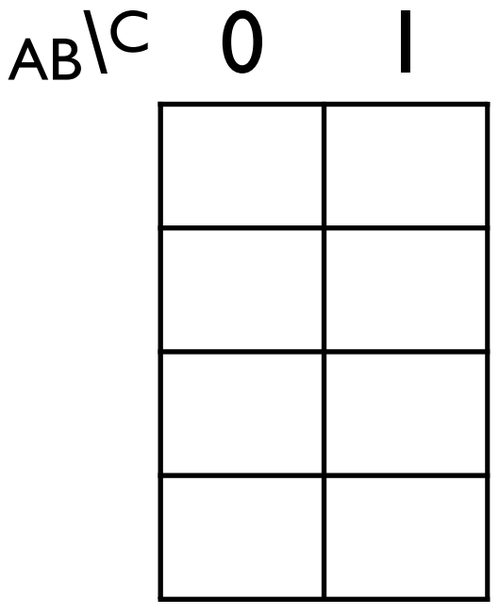
$\uparrow$

$A\,\overline{C}$

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0

- Mapa:



$$Y = \overline{A}\,C + B\,\overline{C} + A\,\overline{B}$$

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0

- Mapa:

AB\C	0	1
00		1
01	1	1
11	1	
10	1	1

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0

- Mapa:

AB\C	0	1
00		1
01	1	1
11	1	
10	1	1

Sempre serão:

$2^1 \rightarrow 2$ células $\rightarrow 1$ var. eliminada

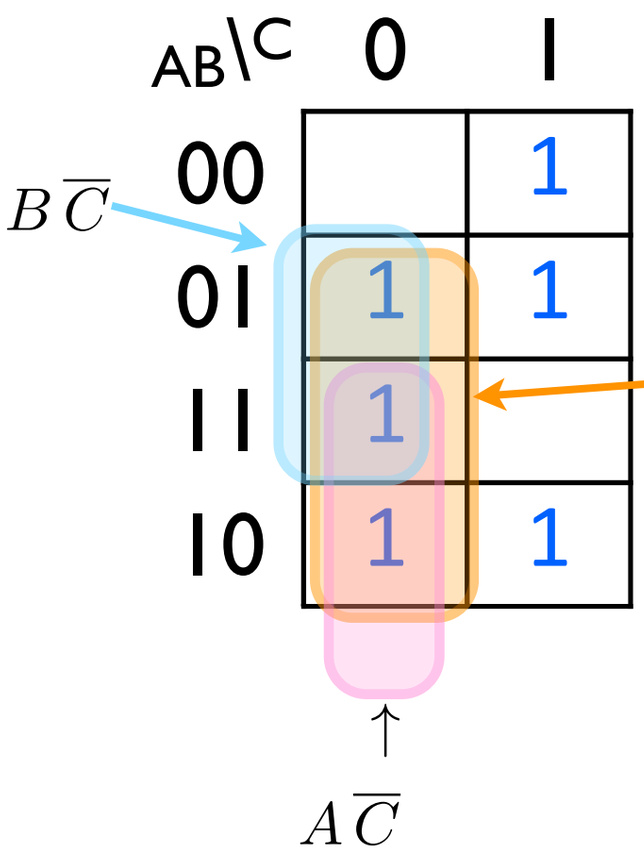
$2^2 \rightarrow 4$ células $\rightarrow 2$ var. eliminadas

$2^3 \rightarrow 8$ células $\rightarrow 3$ var. eliminadas

Detalhe: não existem agrupamentos de 3, 5 células ou os que não sejam múltiplos de $\mathbf{2^n}$.

Exemplo_2:

Tentativa de agrupamento de 3 células: Não resulta!



$$= \overline{A}\,B\,\overline{C} + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C}$$
$$= B\,\overline{C}(\overline{A} + A) + A\,\overline{B}\,\overline{C}$$
$$= B\,\overline{C} + A\,\overline{B}\,\overline{C}$$

Exemplo_2:

AB\C	0	1
00		1
01	1	1
11	1	
10	1	1

Tentativa de agrupamento de 3 células:
Não resulta (nenhuma simplificação direta)!

$$= \overline{A}\,B\,\overline{C} + A\,B\,\overline{C} + A\,\overline{B}\,\overline{C}$$

Desenvolvendo...

ou

$$= B\,\overline{C}\,(\underbrace{\overline{A} + A}_{=1}) + A\,\overline{B}\,\overline{C}$$

$$= B\,\overline{C} + A\,\overline{B}\,\overline{C}$$

$$= \overline{C}(B + A\,\overline{B})$$

$$x + \overline{x}\,y = x + y$$

$$= \overline{C}\,(A + B)$$

$$= \overline{A}\,B\,\overline{C} + A\,\overline{C}(\underbrace{B + \overline{B}}_{=1})$$

$$= \overline{A}\,B\,\overline{C} + A\,\overline{C}$$

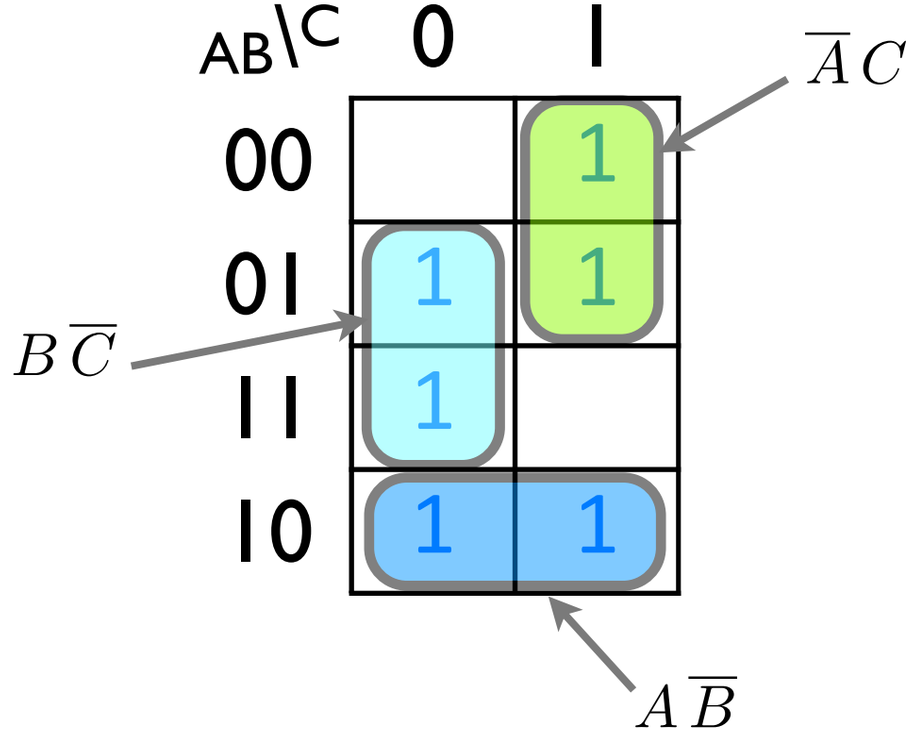
$$= \overline{C}\,(\overline{A}\,B + A)$$

$$= \overline{C}\,(A + B)$$

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0

- Mapa:

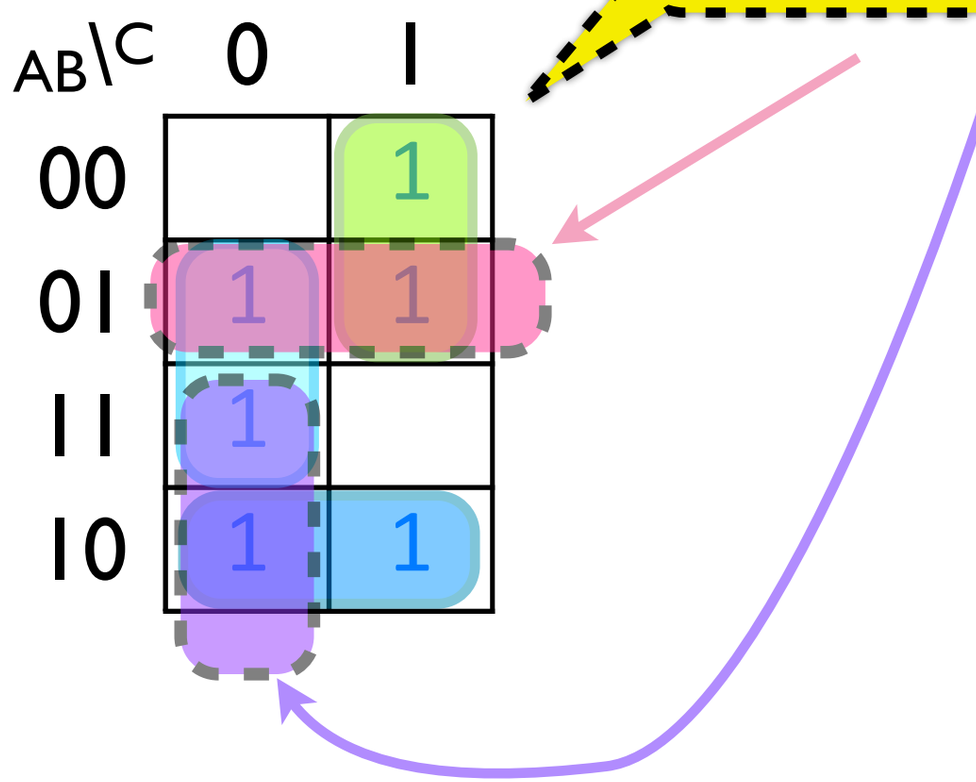


$$Y = \overline{A}\,C + B\,\overline{C} + A\,\overline{B}$$

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0

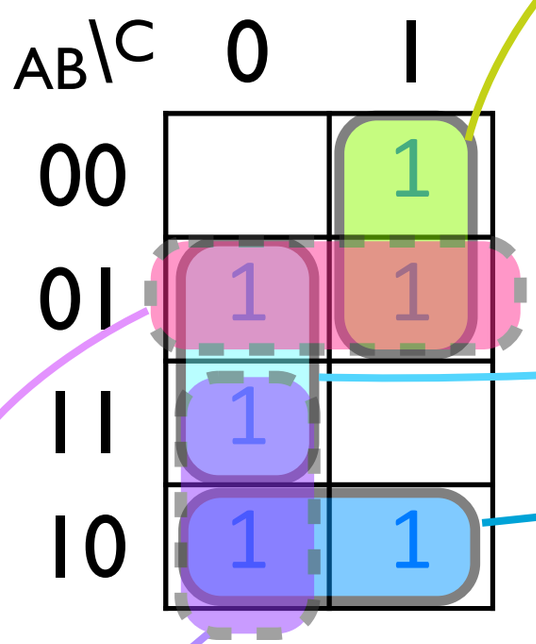
- Mapa:

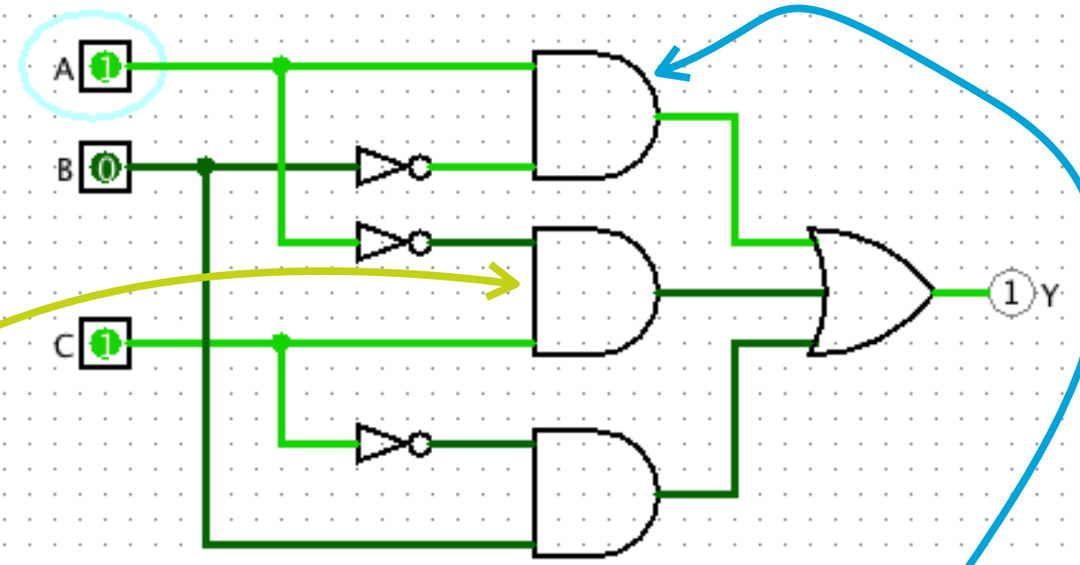


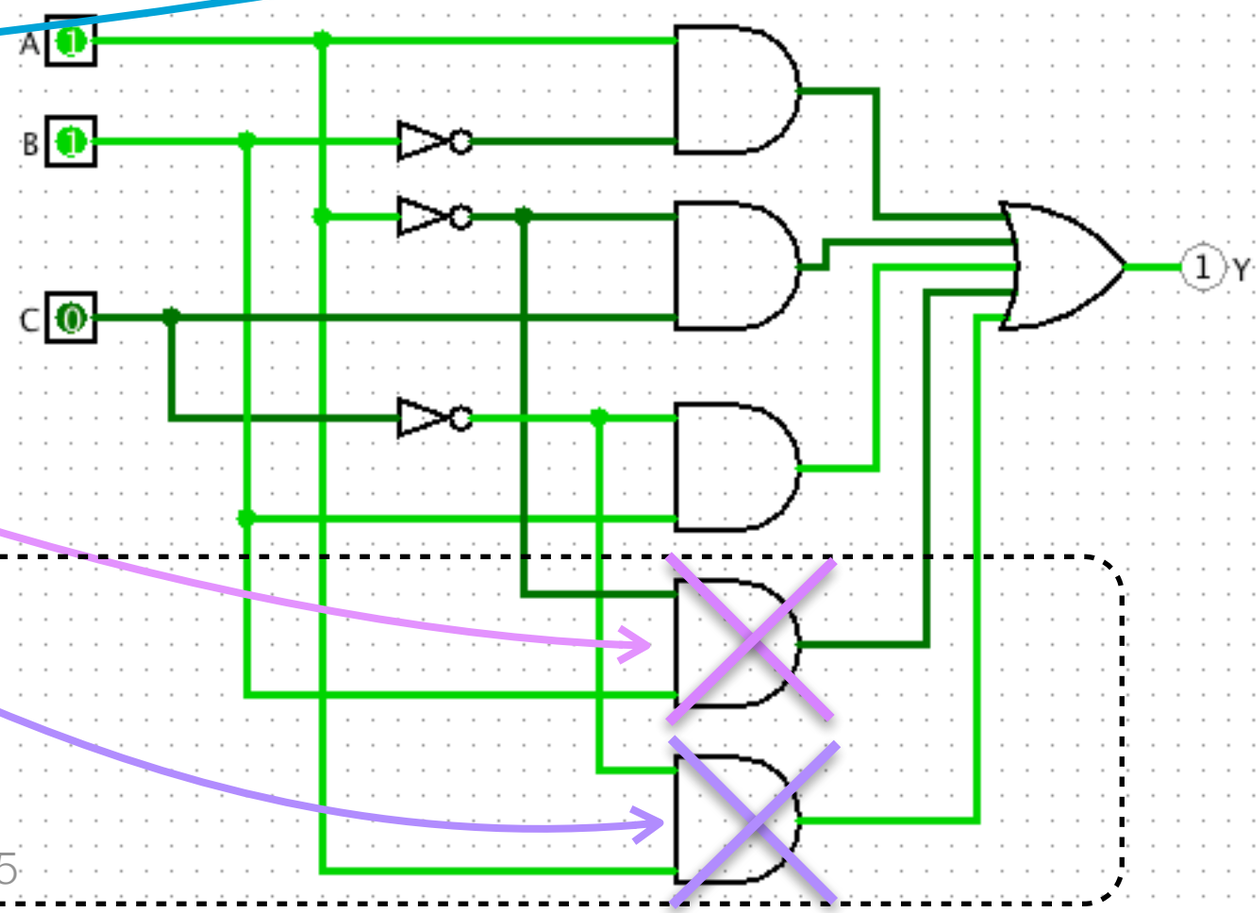
- Não implica em erro, mas aumenta circuito!

Exemplo_2:

Ref	ABC	Y
0	000	0
1	001	1
2	010	1
3	011	1
4	100	1
5	101	1
6	110	1
7	111	0







Agrupamentos Redundantes:
Não implica em erro, mas aumenta circuito!
(Portas desnecessárias)

Mapa K para 4 variáveis

Ref	ABCD	Y
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

- Mapa - Opção 1:

AB\CD	00	01	11	10
00				
01				
11				
10				

- Mapa - Opção 2:

CD\AB	00	01	11	10
00				
01				
11				
10				

Mapa K para 4 variáveis

Ref	ABCD	Y
0	0000	m_0
1	0001	m_1
2	0010	m_2
3	0011	m3
4	0100	m4
5	0101	m5
6	0110	m6
7	0111	m7
8	1000	m8
9	1001	m9
10	1010	m10
11	1011	m11
12	1100	m12
13	1101	m13
14	1110	m14
15	1111	m15

- Mapa - Opção 1:

AB\CD	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

- Mapa - Opção 2:

CD\AB	00	01	11	10
00	m0	m4	m_{12}	m8
01	m1	m5	m_{13}	m9
11	m3	m7	m_{15}	m_{11}
10	m2	m6	m_{14}	m_{10}

Mapa K para 4 variáveis

Ref	ABCD	Y
0	0000	m_0
1	0001	m_1
2	0010	m_2
3	0011	m_3
4	0100	m_4
5	0101	m_5
6	0110	m_6
7	0111	m_7
8	1000	m_8
9	1001	m_9
10	1010	m_{10}
11	1011	m_{11}
12	1100	m_{12}
13	1101	m_{13}
14	1110	m_{14}
15	1111	m_{15}

- Mapa - Opção 1:

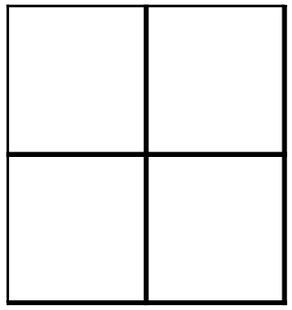
$AB\backslash CD$	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Problemas

Obs.: Tente resolver

Ref	A	B	X
0	0	0	1
1	0	1	0
2	1	0	0
3	1	1	1

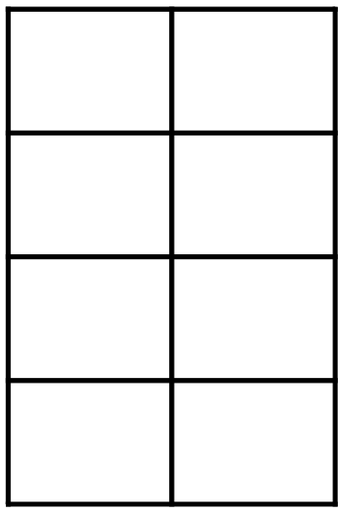
A\B



$$F = \bar{A}\bar{B} + AB$$

Ref	ABC	Y
0	000	1
1	001	1
2	010	1
3	011	0
4	100	0
5	101	0
6	110	1
7	111	0

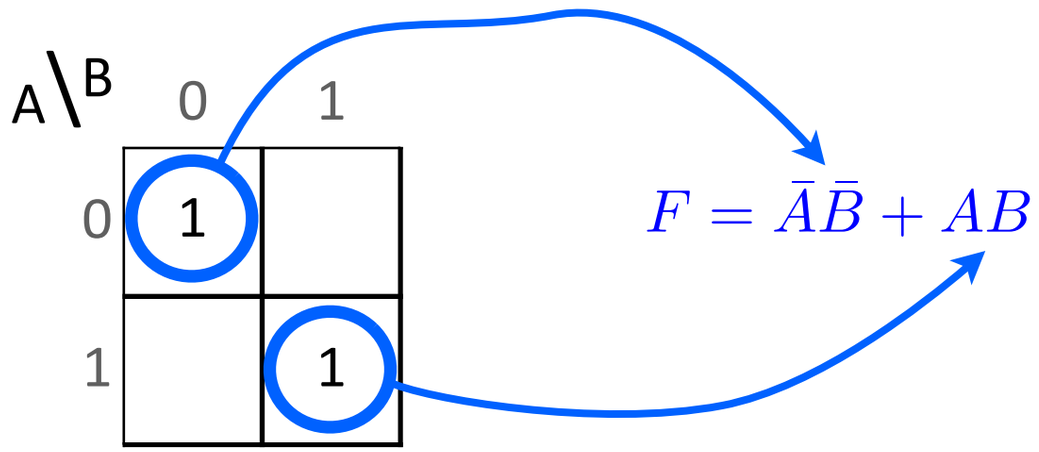
AB\C



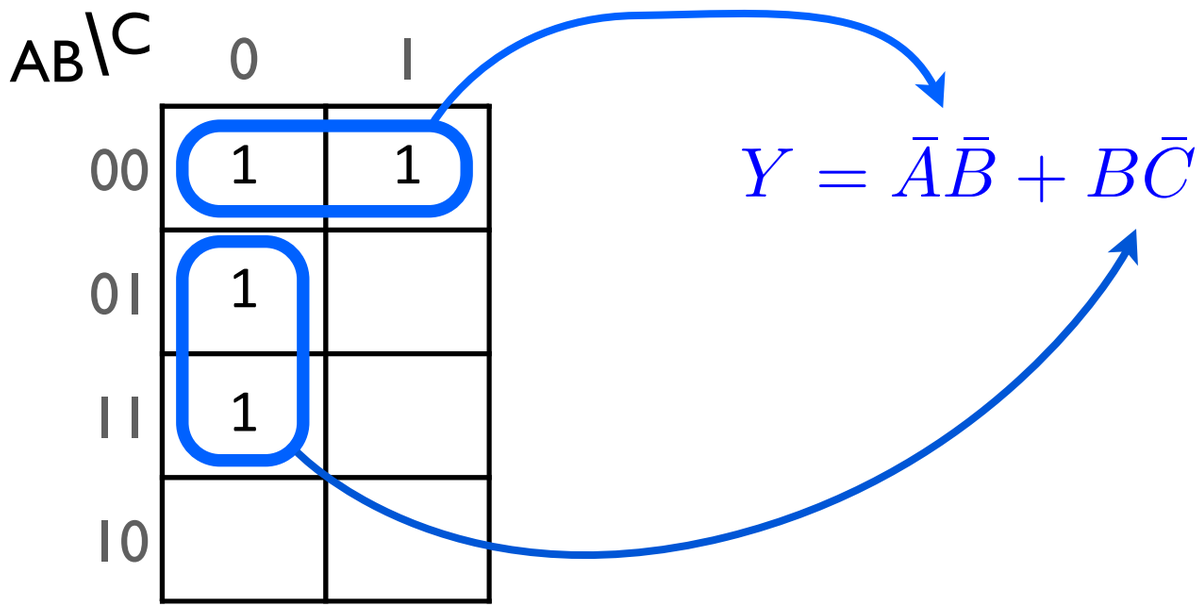
$$Y = \bar{A}\bar{B} + B\bar{C}$$

Soluções

Ref	A	B	X
0	0	0	1
1	0	1	0
2	1	0	0
3	1	1	1

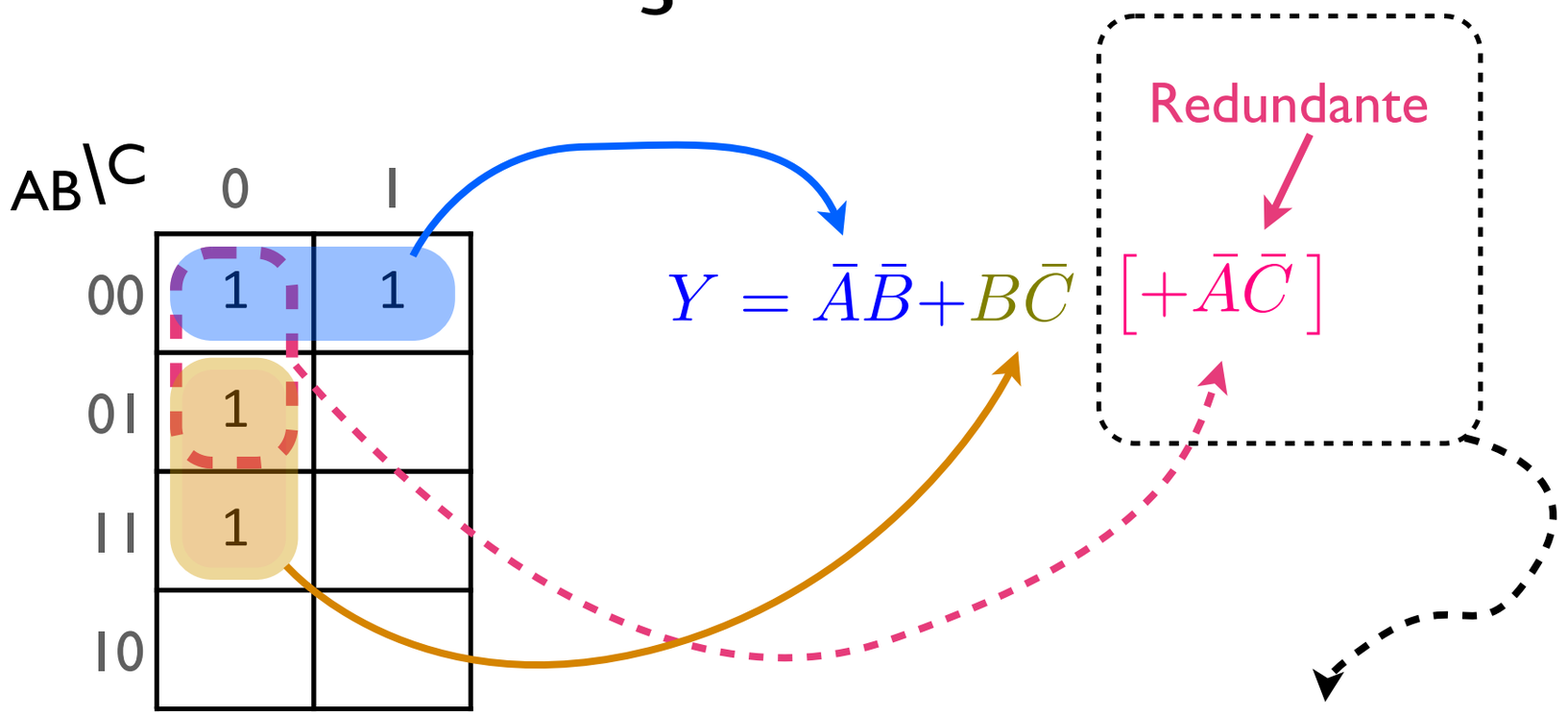


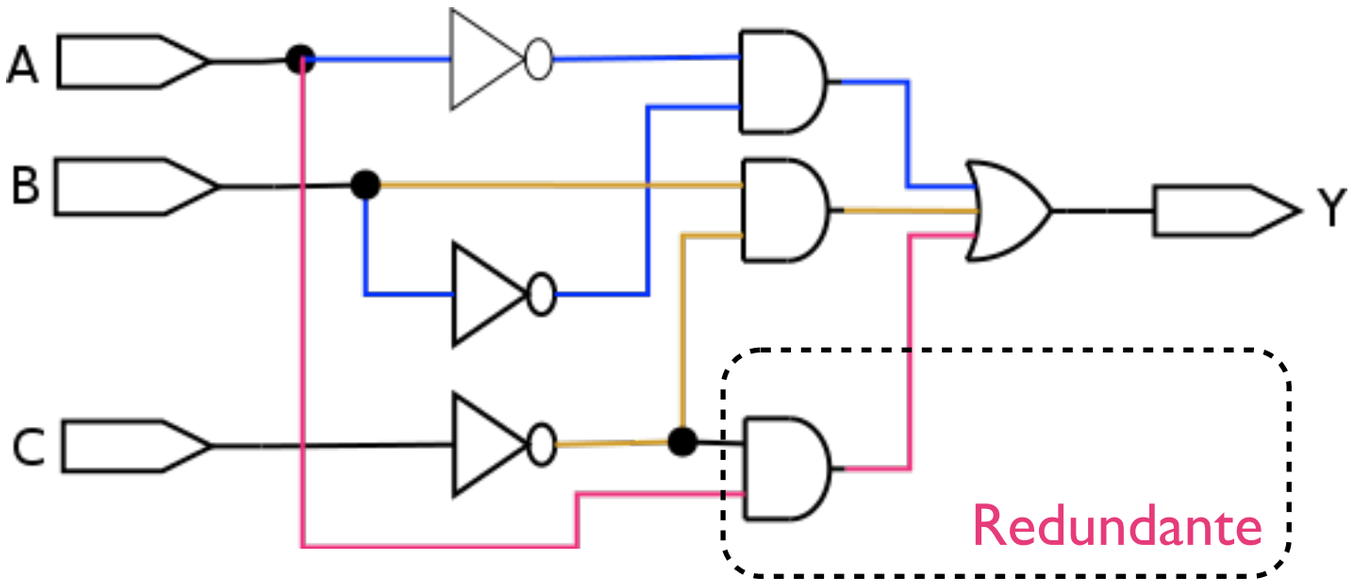
Ref	ABC	Y
0	000	1
1	001	1
2	010	1
3	011	0
4	100	0
5	101	0
6	110	1
7	111	0



Observações

Ref	ABC	Y
0	000	1
1	001	1
2	010	1
3	011	0
4	100	0
5	101	0
6	110	1
7	111	0

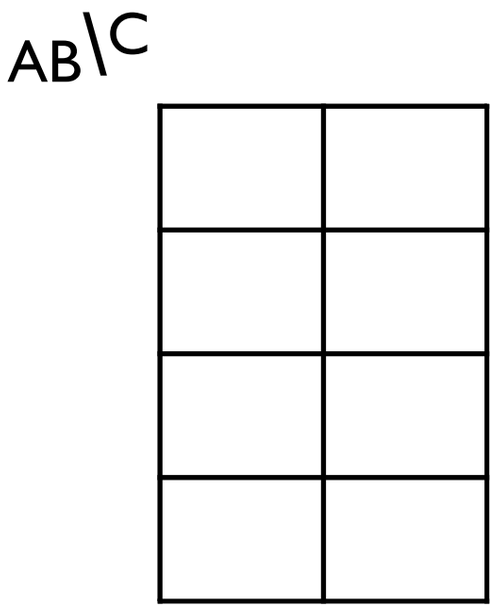




Ref	A B C	Y
0	0 0 0	1
1	0 0 1	1
2	0 1 0	1
3	0 1 1	0
4	1 0 0	0
5	1 0 1	0
6	1 1 0	1
7	1 1 1	0

Problema:

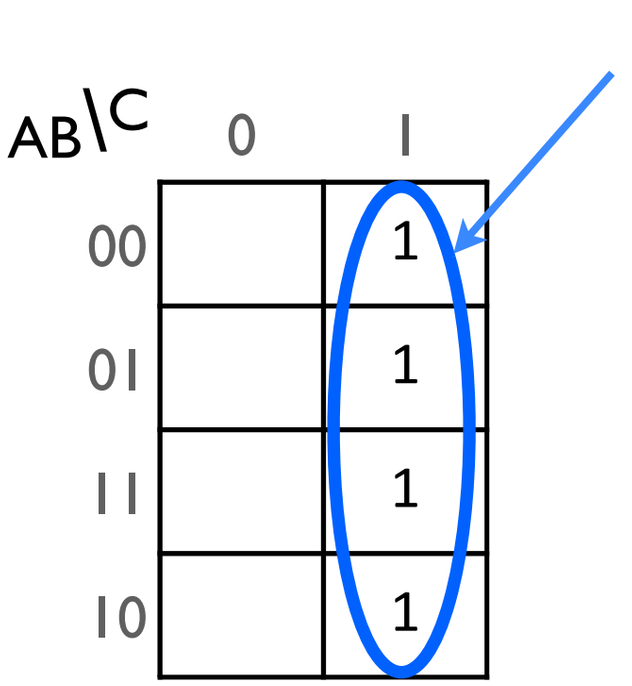
Ref	ABC	Y
0	000	0
1	001	1
2	010	0
3	011	1
4	100	0
5	101	1
6	110	0
7	111	1



Resp.: $Y = C$

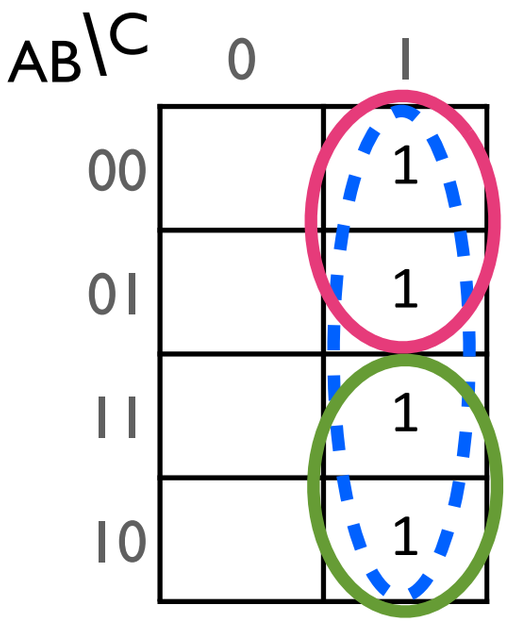
Solução:

Ref	ABC	Y
0	000	0
1	001	1
2	010	0
3	011	1
4	100	0
5	101	1
6	110	0
7	111	1



Prova:

$$Y = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$
$$Y = \bar{A}C(\bar{B} + B) + AC(\bar{B} + B)$$
$$Y = \bar{A}C + AC$$
$$Y = C(\bar{A} + A)$$
$$Y = C$$

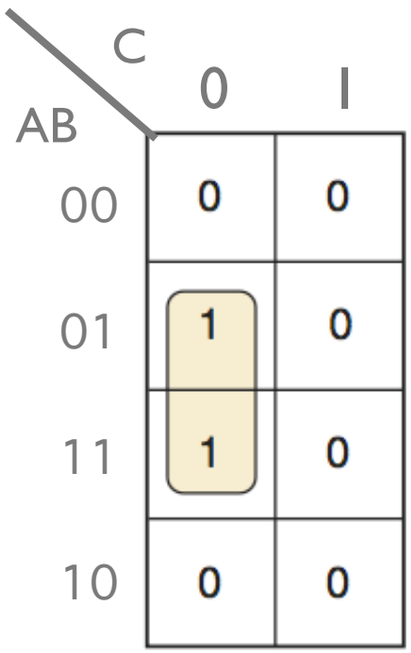


$$Y = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$
$$Y = \bar{A}C(\bar{B} + B) + AC(\bar{B} + B)$$
$$Y = \bar{A}C + AC$$
$$Y = C(\bar{A} + A)$$
$$Y = C$$

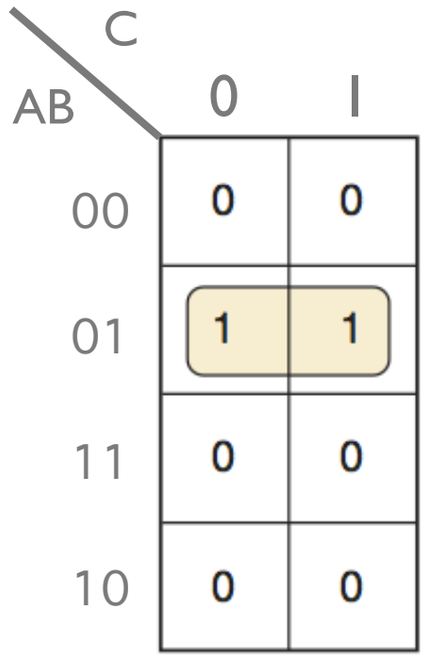
Agrupamentos Possíveis

- Com 2 variáveis

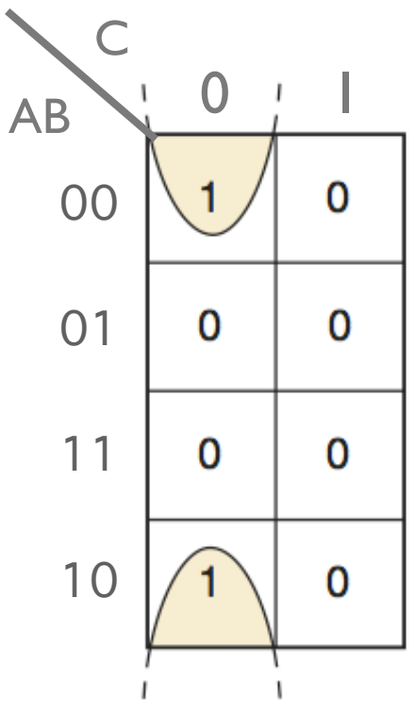
Obs.: Tente resolver:



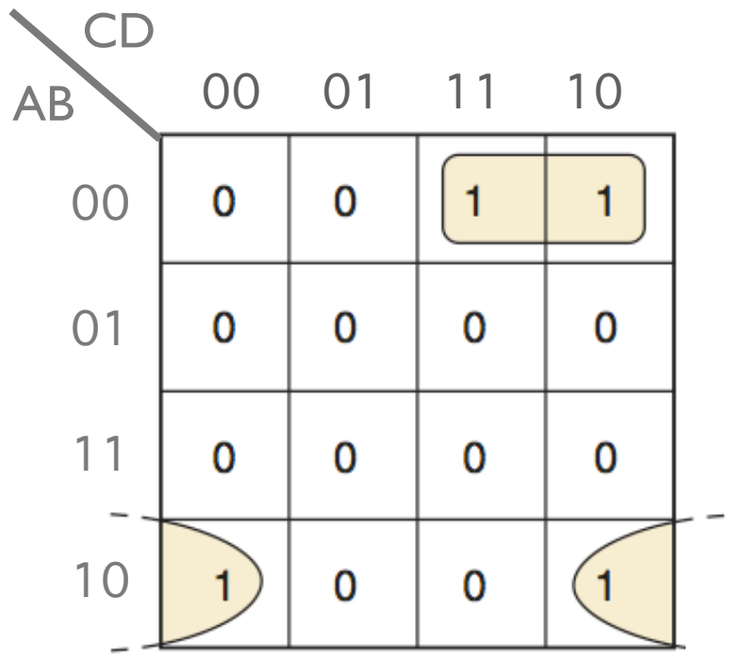
(a)



(b)



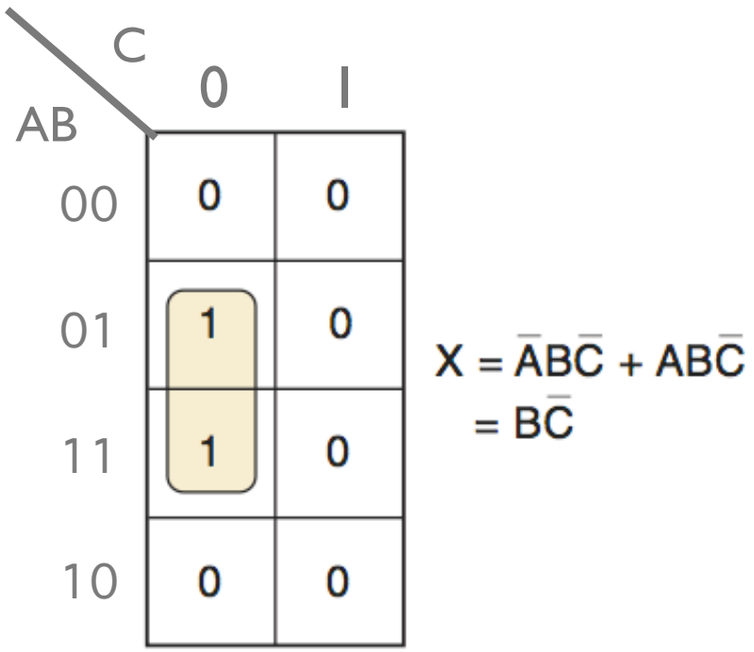
(c)



(d)

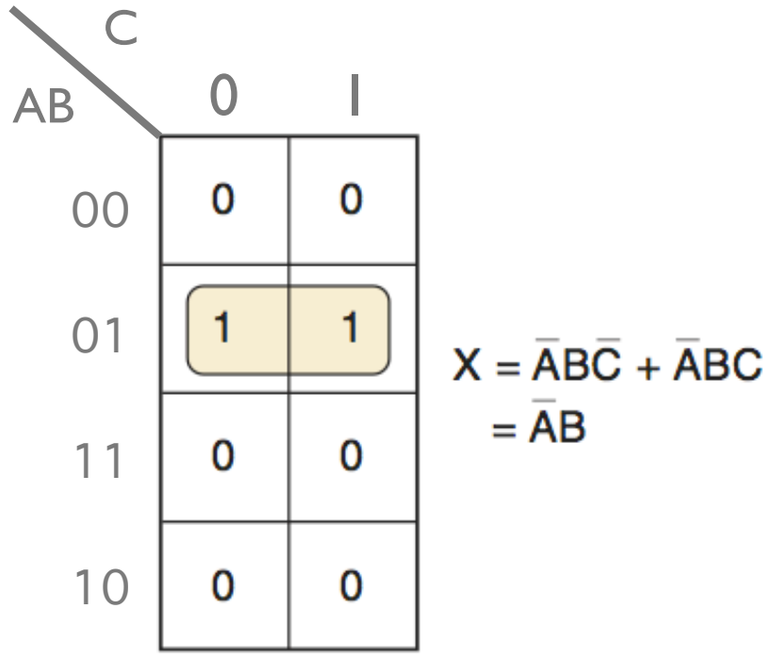
Agrupamentos Possíveis

- Com 2 variáveis



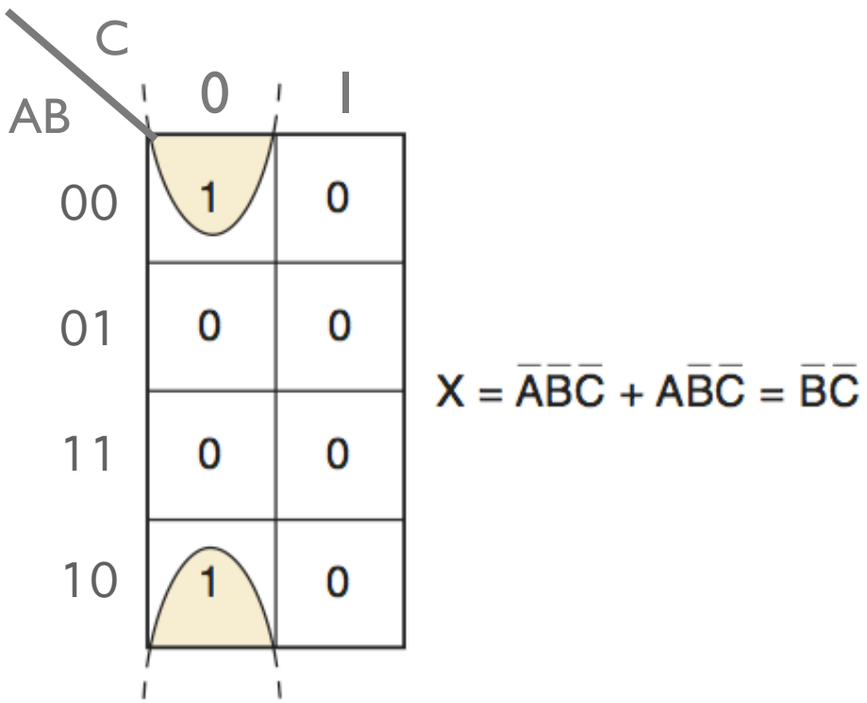
$X = \bar{A}B\bar{C} + AB\bar{C}$
$= B\bar{C}$

(a)



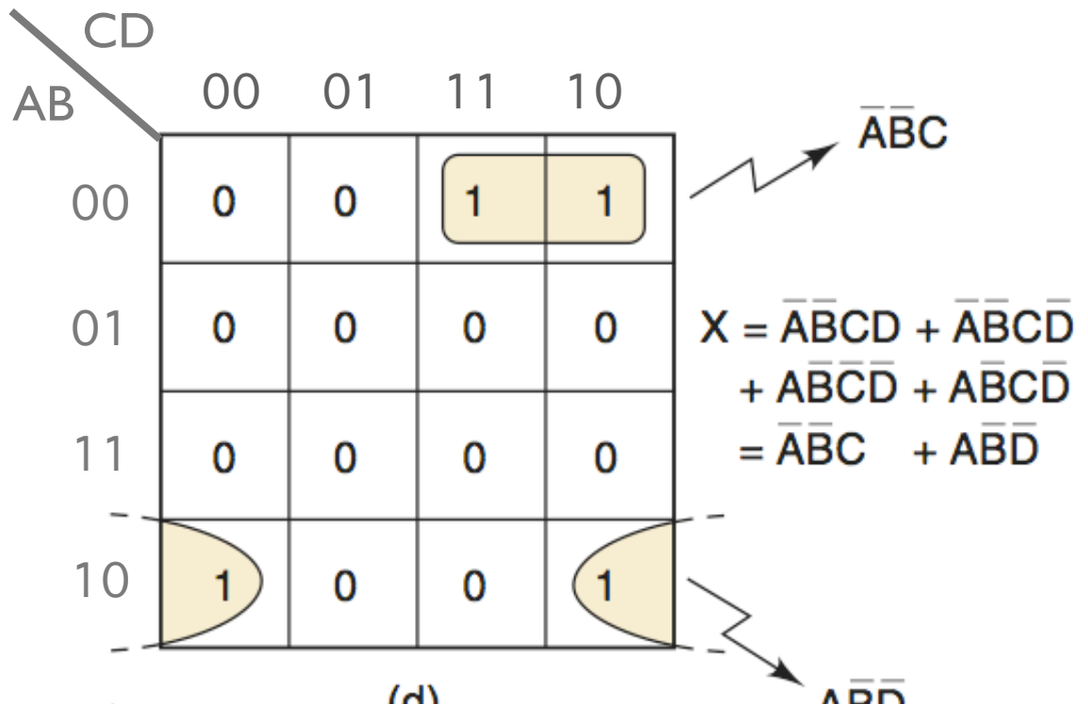
$X = \bar{A}B\bar{C} + \bar{A}BC$
$= \bar{A}B$

(b)



$X = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} = \bar{B}\bar{C}$

(c)



$X = \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D}$
$+ A\bar{B}\bar{C}D + A\bar{B}C\bar{D}$
$= \bar{A}\bar{B}C + A\bar{B}\bar{D}$

(d)

Agrupamentos Possíveis

- Com 4 variáveis

Obs.: Tente resolver:

(a)

AB \ C	0	1
00	0	1
01	0	1
11	0	1
10	0	1

(b)

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

(c)

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

(d)

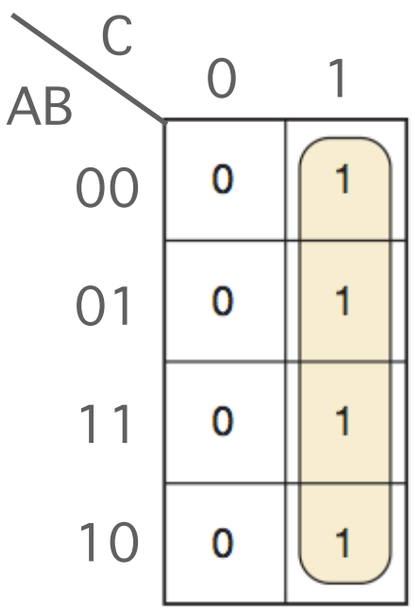
AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	0	0	1
10	1	0	0	1

(e)

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

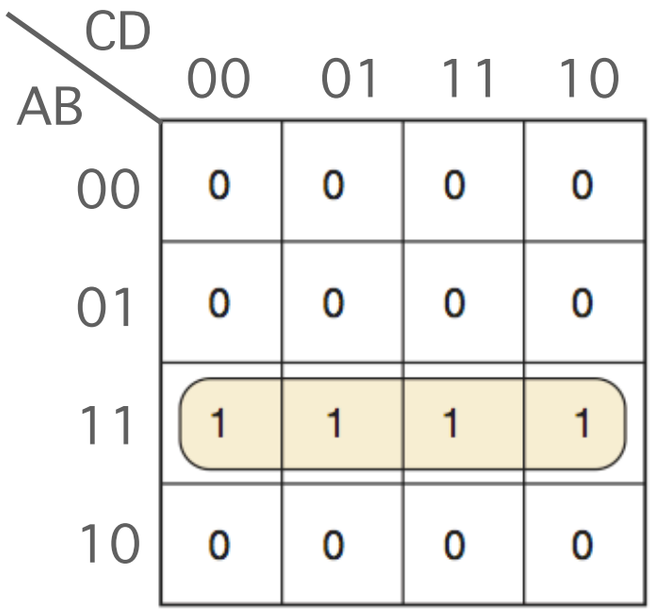
Agrupamentos Possíveis

- Com 4 variáveis



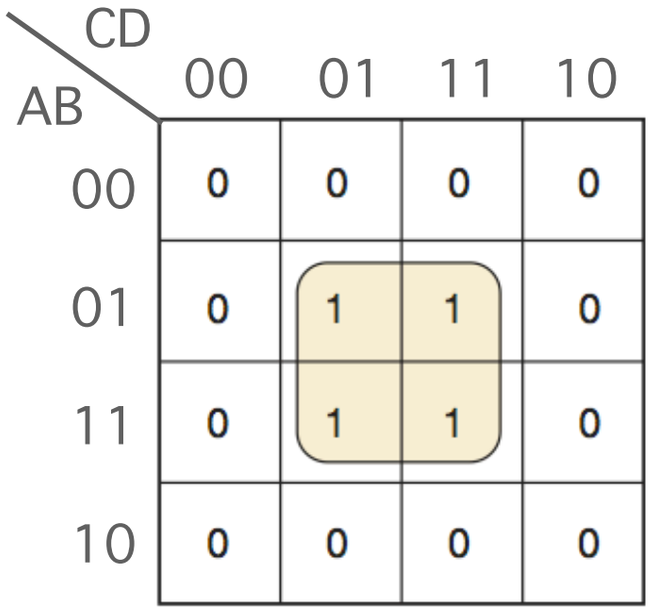
X = C

(a)



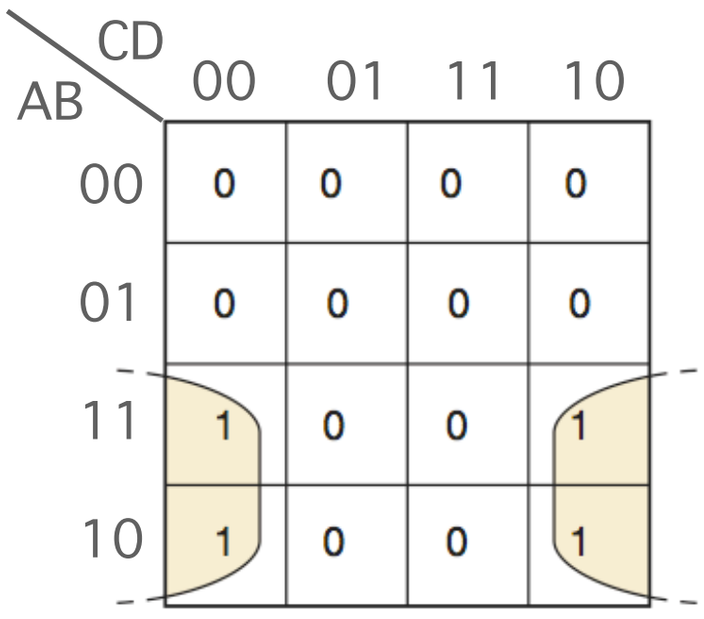
X = AB

(b)



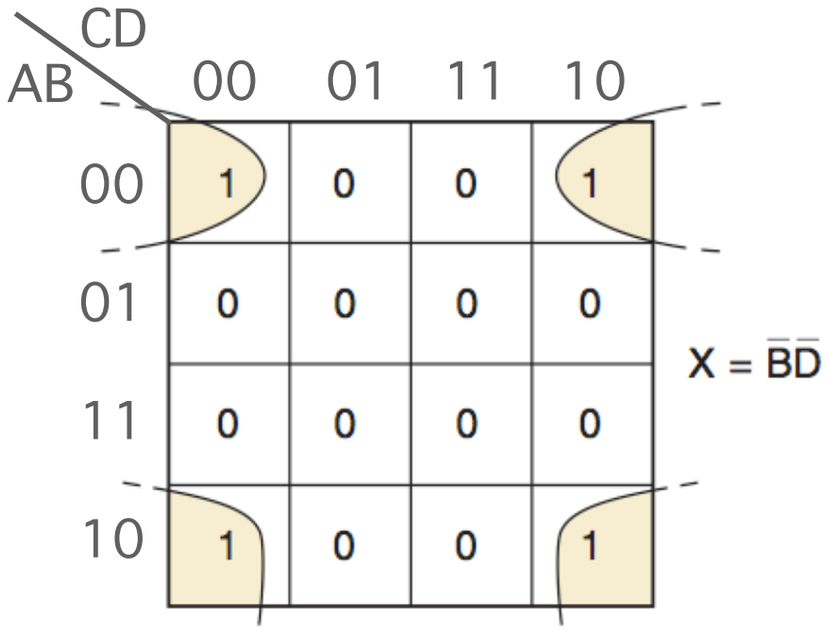
X = BD

(c)



X = $A\bar{D}$

(d)



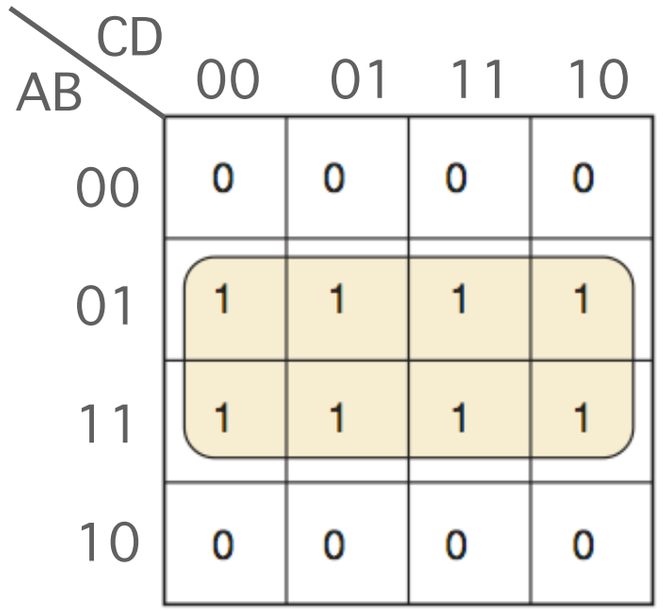
X = $\bar{B}\bar{D}$

(e)

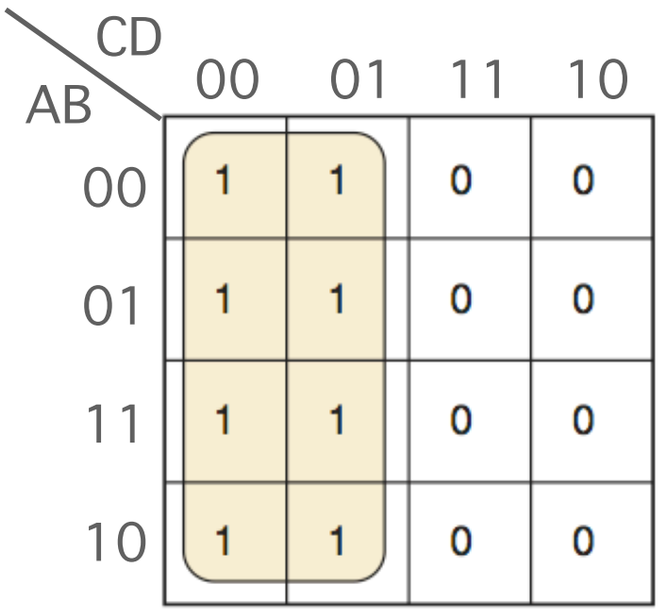
Agrupamentos Possíveis

- Com 8 células

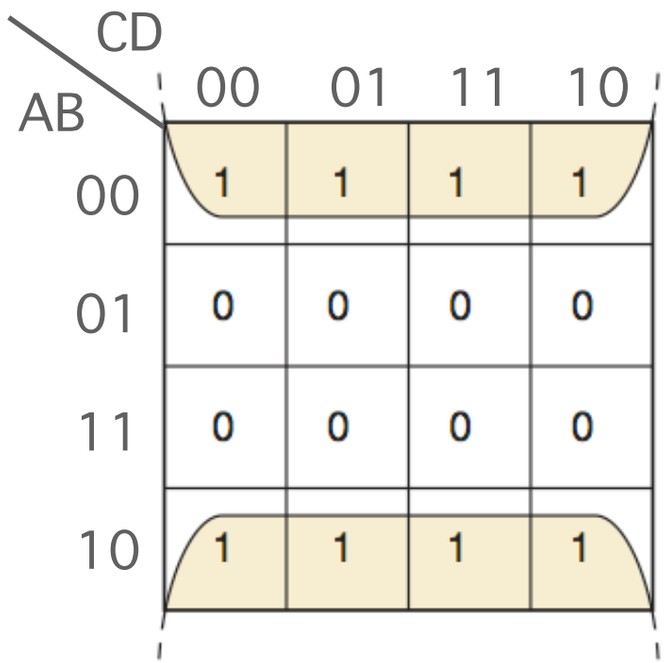
Obs.: Tente resolver:



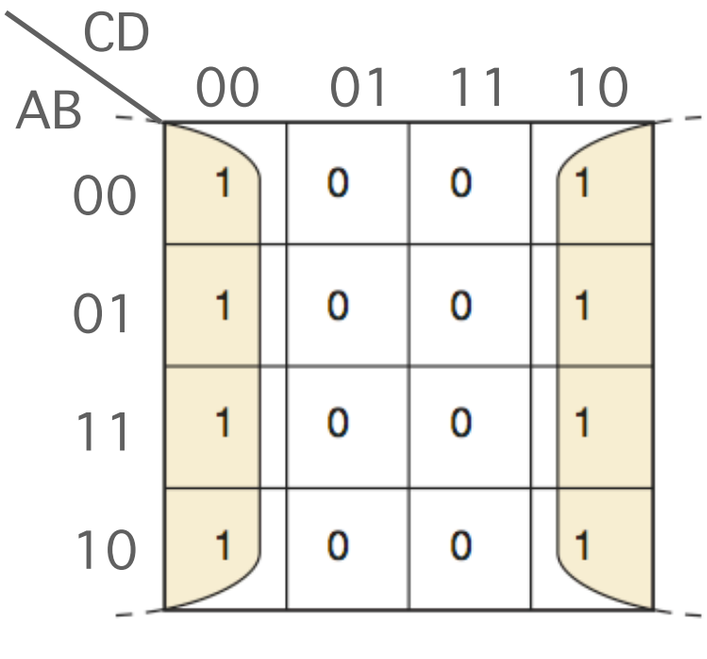
(a)



(b)



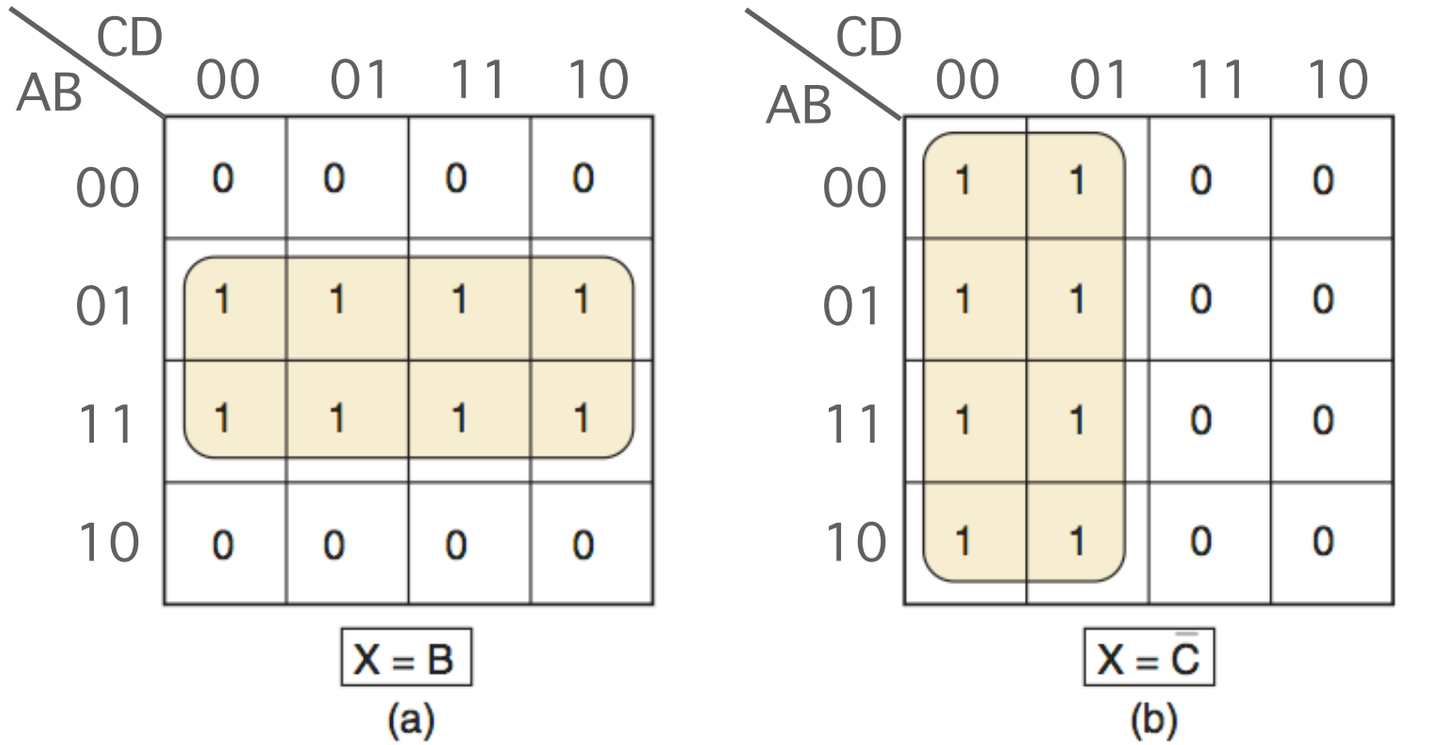
(c)

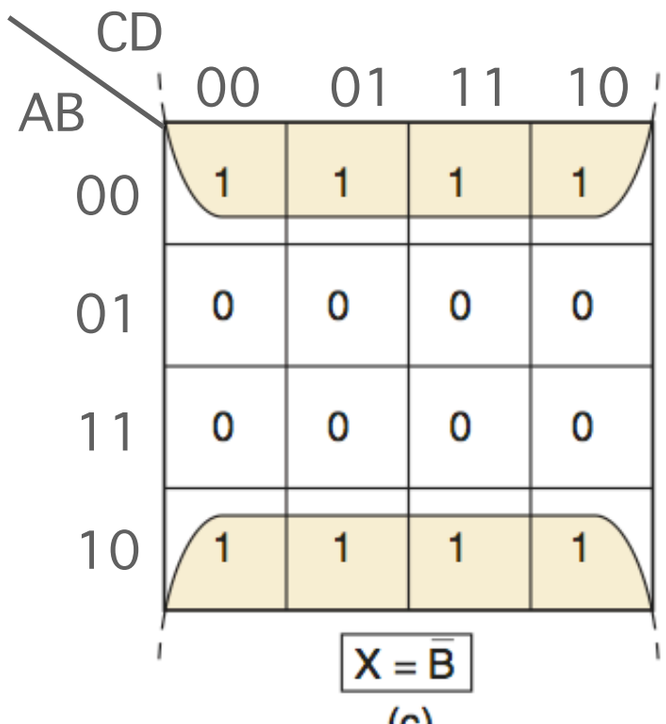


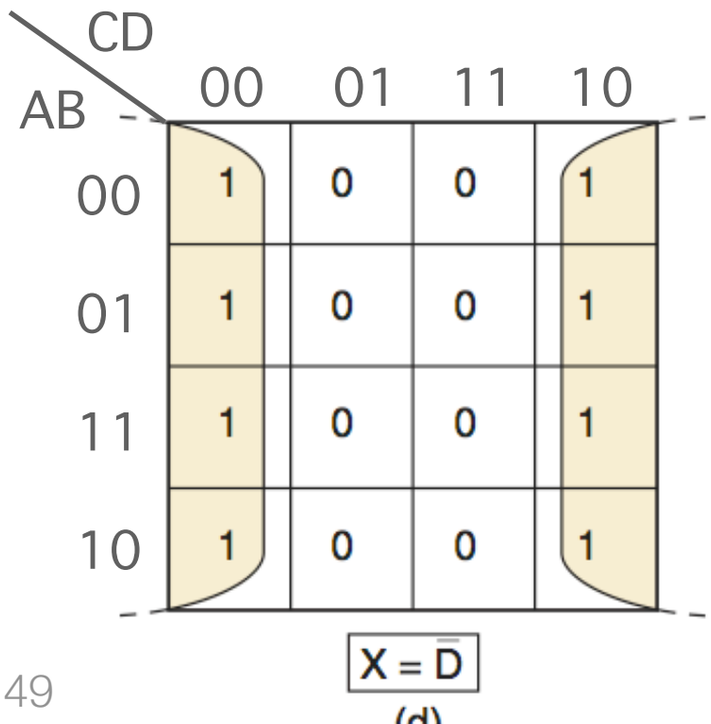
(d)

Agrupamentos Possíveis

- Com 8 células







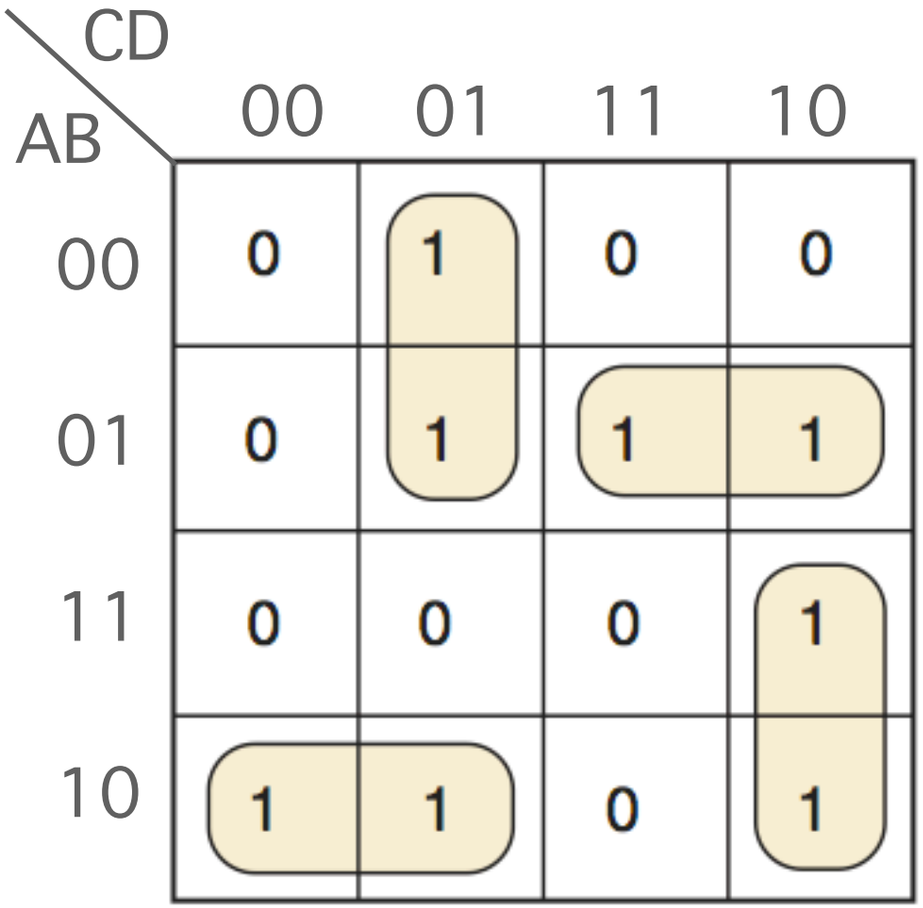
■ Mesmo problema - 2 soluções possíveis:

Obs.: Tente resolver:

CD / AB	00	01	11	10
00		1		
01		1	1	1
11				1
10	1	1		1

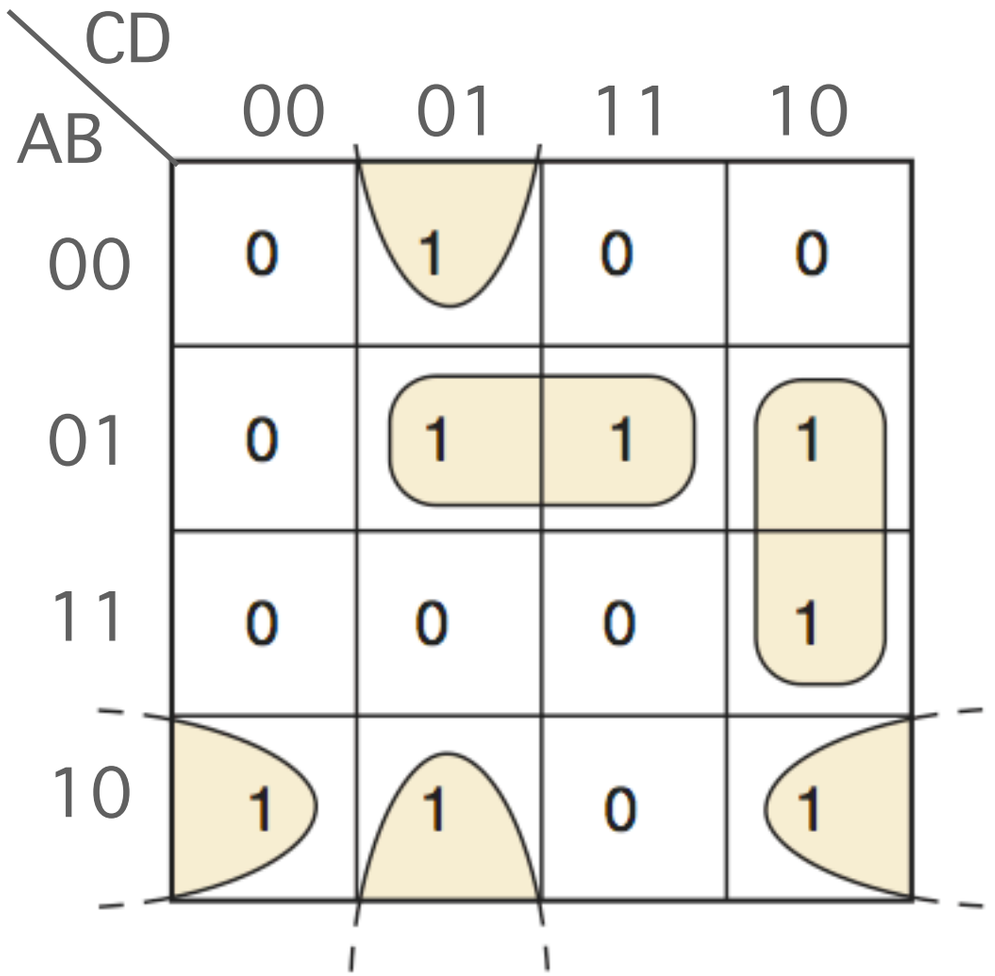
CD / AB	00	01	11	10
00		1		
01		1	1	1
11				1
10	1	1		1

Mesmo problema - 2 soluções possíveis:



$$X = \bar{A}\bar{C}D + \bar{A}BC + A\bar{B}\bar{C} + AC\bar{D}$$

(a)



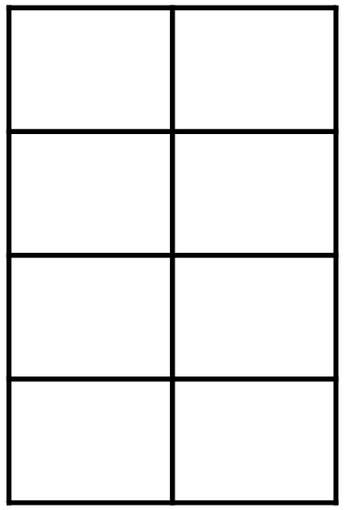
$$X = \bar{A}BD + BC\bar{D} + \bar{B}\bar{C}D + A\bar{B}\bar{D}$$

(b)

Problema

Ref	ABC	Y
0	000	1
1	001	1
2	010	1
3	011	0
4	100	1
5	101	1
6	110	1
7	111	0

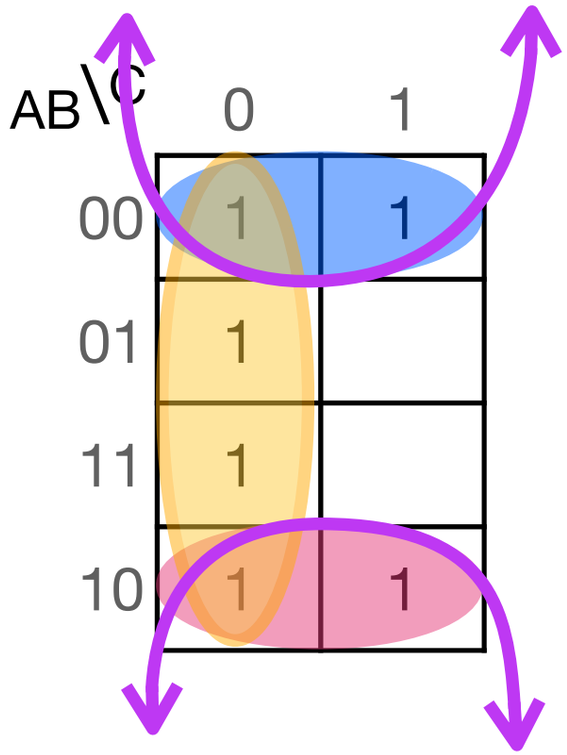
AB\C



Resp.: $Y = \bar{B} + \bar{C}$

Solução

Ref	ABC	Y
0	000	1
1	001	1
2	010	1
3	011	0
4	100	1
5	101	1
6	110	1
7	111	0



$$Y = \bar{C} + \bar{A}\bar{B} + A\bar{B}$$

$$Y = \bar{C} + \bar{B}(\bar{A} + A)$$

$$Y = \bar{C} + \bar{B}$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

$$Y = \bar{A}\bar{B}(\bar{C} + C) + B\bar{C}(\bar{A} + A) + A\bar{B}(\bar{C} + C)$$

$$Y = \bar{A}\bar{B} + B\bar{C} + A\bar{B}$$

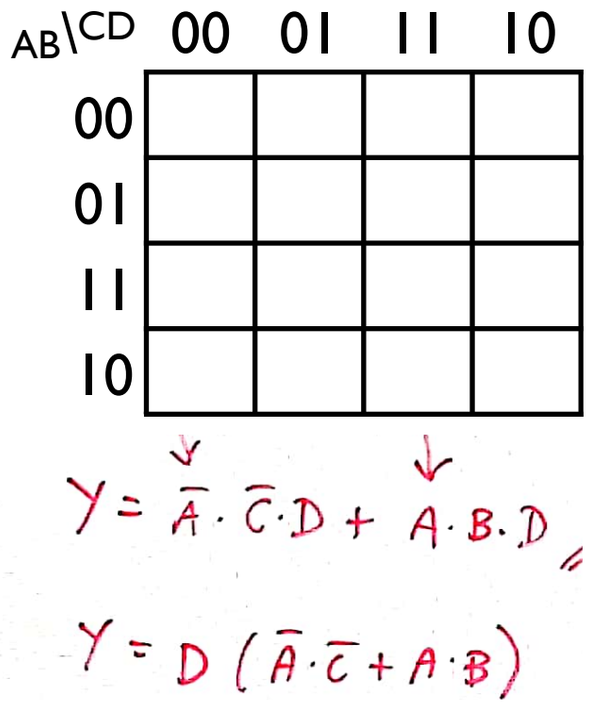
$$Y = \bar{B}(\bar{A} + A) + B\bar{C}$$

$$Y = \bar{B} + B\bar{C}$$

$$Y = \bar{B} + \bar{C}$$

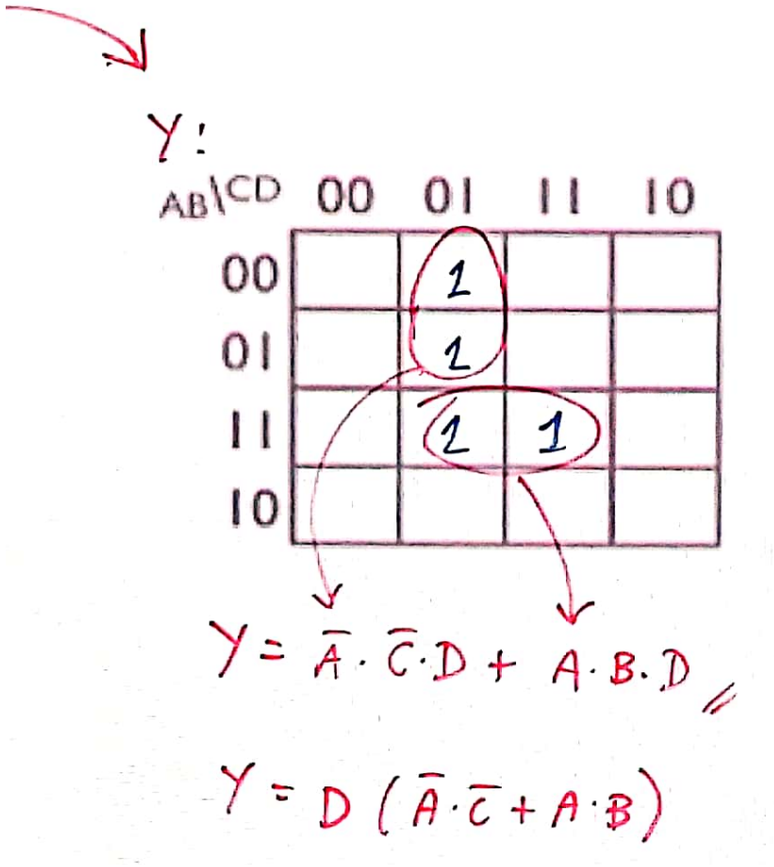
Problemas

Ref	ABCD	Y
0	0000	0
1	0001	1
2	0010	0
3	0011	0
4	0100	0
5	0101	1
6	0110	0
7	0111	0
8	1000	0
9	1001	0
10	1010	0
11	1011	0
12	1100	0
13	1101	1
14	1110	0
15	1111	1



Problemas

Ref	ABCD	Y
0	0000	0
1	0001	1
2	0010	0
3	0011	0
4	0100	0
5	0101	1
6	0110	0
7	0111	0
8	1000	0
9	1001	0
10	1010	0
11	1011	0
12	1100	0
13	1101	1
14	1110	0
15	1111	1



Problemas

AB\CD	00	01	11	10
00				1
01		1	1	
11		1	1	
10			1	

AB\CD	00	01	11	10
00			1	
01	1	1	1	1
11	1	1		
10				

AB\CD	00	01	11	10
00		1	1	1
01	1	1	1	1
11				
10				

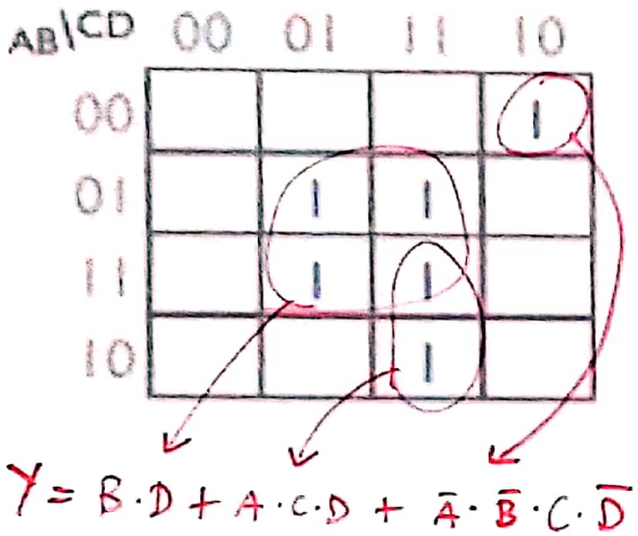
AB\CD	00	01	11	10
00		1		
01		1	1	1
11	1	1	1	
10			1	

AB\CD	00	01	11	10
00	1		1	1
01	1			1
11				
10	1		1	1

AB\CD	00	01	11	10
00				
01				
11				
10				

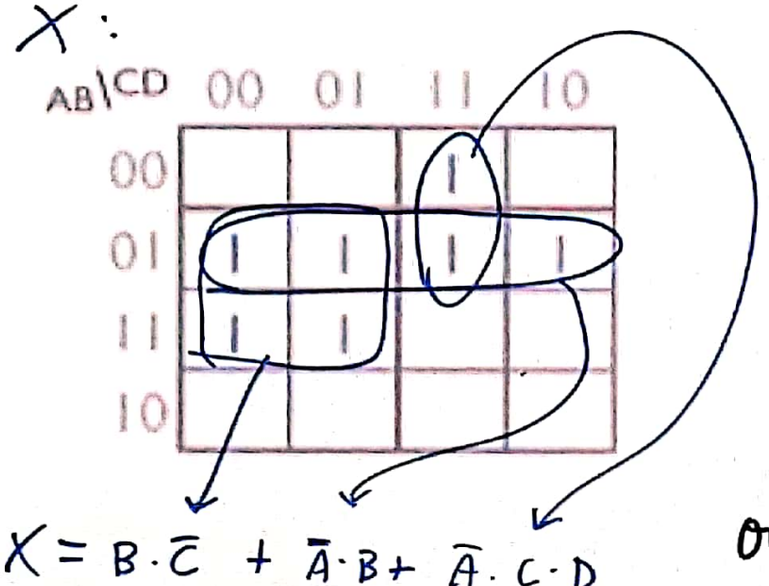
Problemas

Y:



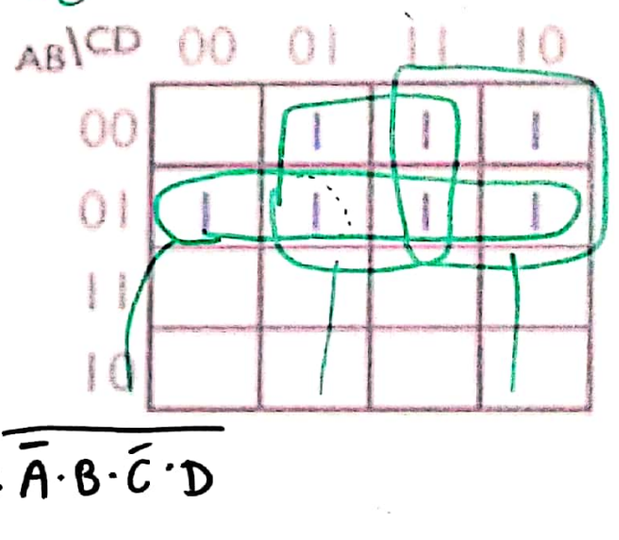
$Y = B \cdot D + A \cdot C \cdot D + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$

X:



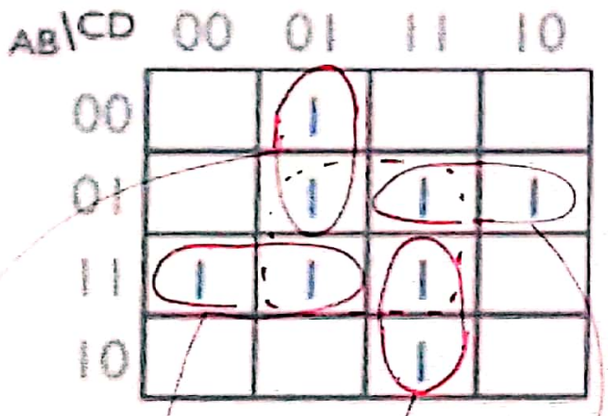
$X = B \cdot \bar{C} + \bar{A} \cdot B + \bar{A} \cdot C \cdot D$

Z:

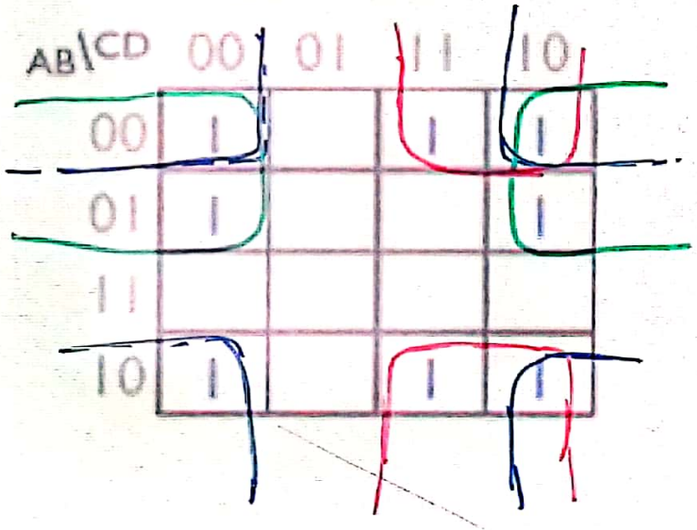


ou $W = \overline{\bar{A} \cdot B \cdot \bar{C} \cdot D}$

Y =

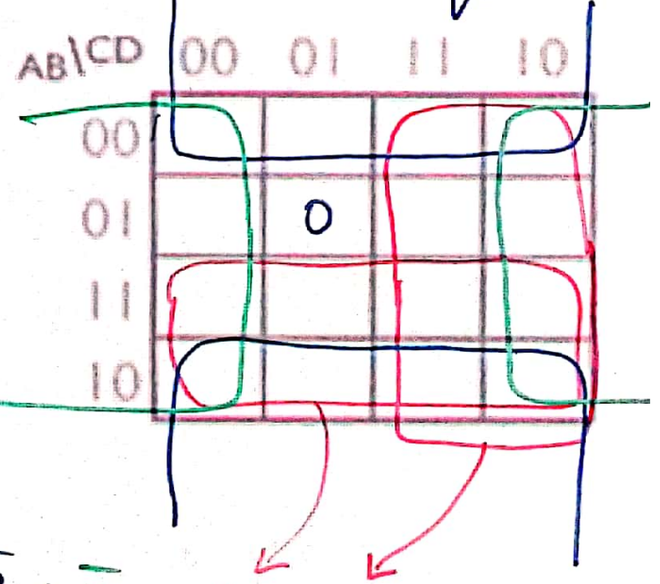


$Y = A \cdot B \cdot \bar{C} + A \cdot C \cdot D + \bar{A} \cdot B \cdot C + \bar{A} \cdot \bar{C} \cdot D$ [+ B.D]



$= A \cdot \bar{D} + \bar{B} \cdot C + \bar{B} \cdot \bar{D}$

ou



$W = \bar{B} + \bar{D} + A + C$

Fim