Sustentador de Ordem Zero
Cálculo de BoG(z)

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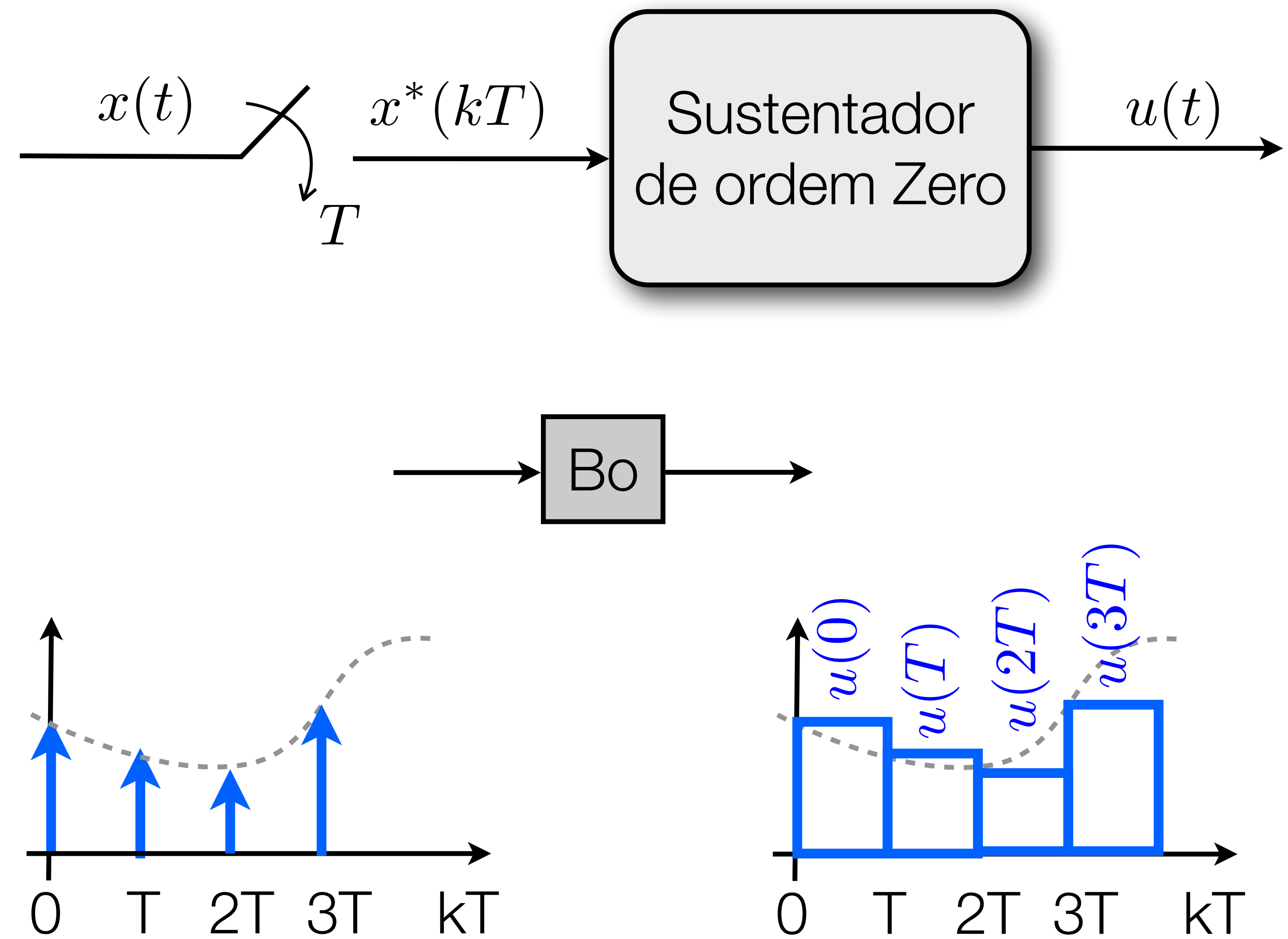
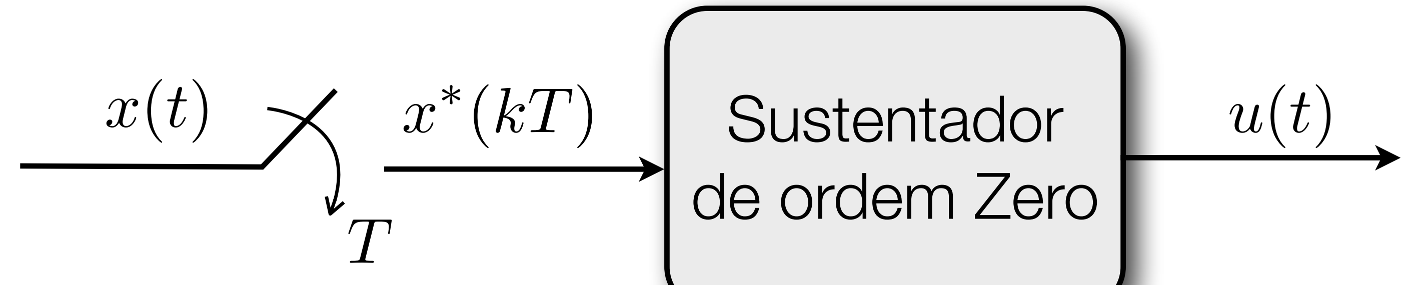
Sistemas com Bloqueador (ou Sustentador ou “Holder”)

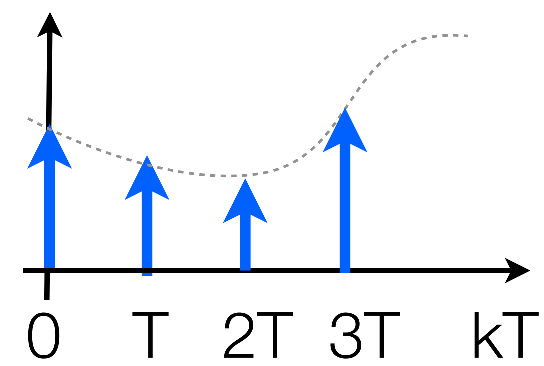
- Sistemas discretos no tempo --> sequencia de números;
- Processo para obter sequencia de números --> amostragem (“*sampling*”);
- Valores de x(t) são tomados em pontos equidistantes de tempo: {x(kT)}, k= 0, 1, 2, 3, ... --> sinal discreto no tempo.
- O sinal discreto é obtido através de uma chave que “congela” este sinal até o próximo instante de amostragem --> “*holder*”.
- Amostragem é ideal se saída da chave amostradora (*sampler*) seja praticamente um degrau --> Bloqueador de ordem zero: Bo.

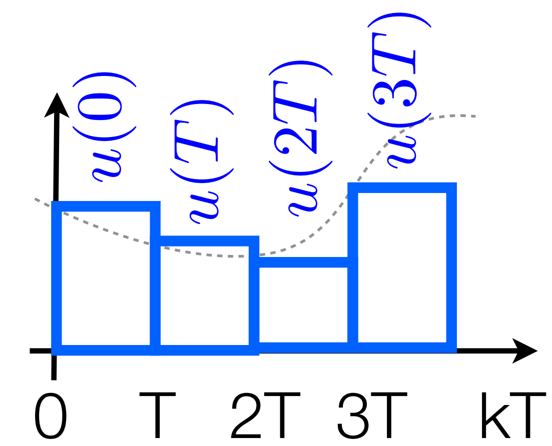


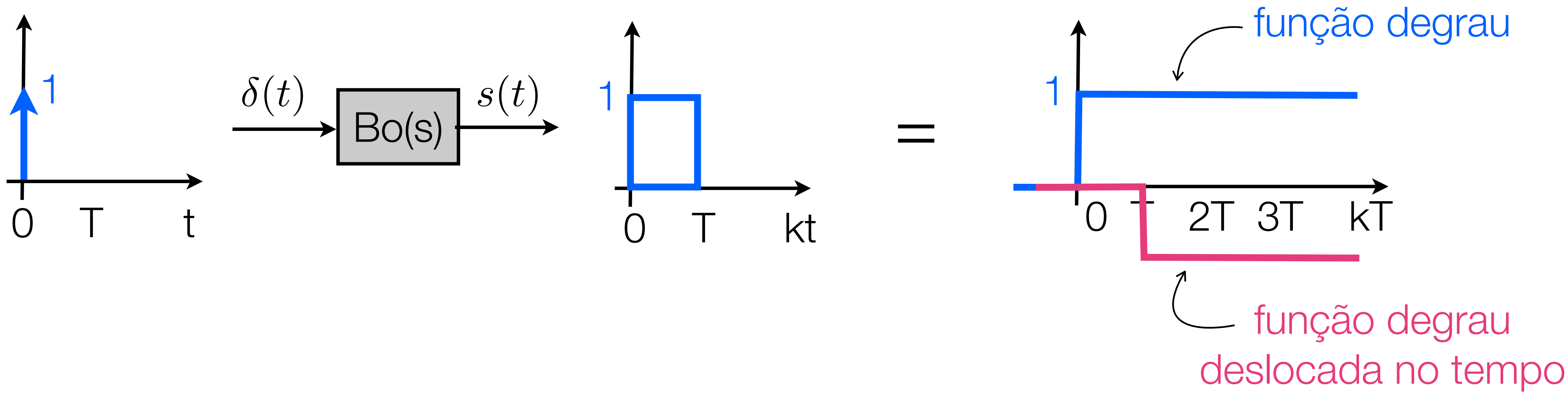
Sistemas com Bloqueador (ou Sustentador ou “Holder”)

- Modelagem:

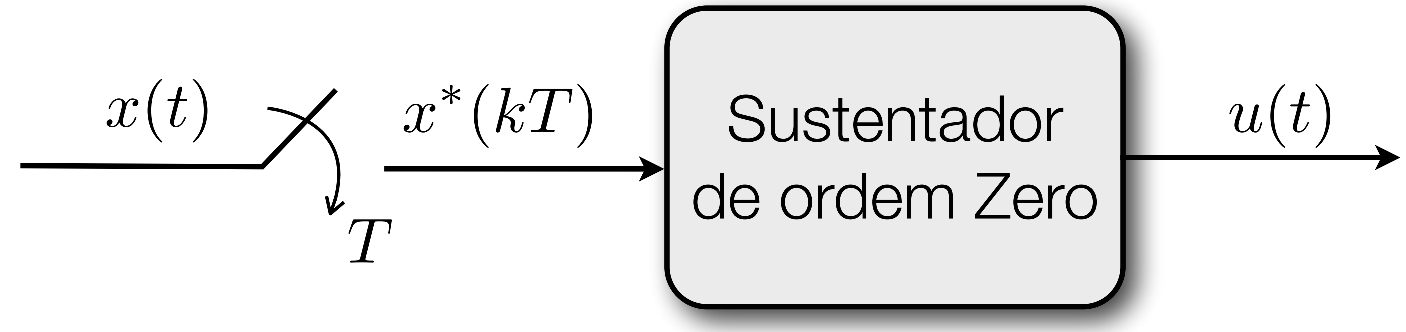








Sistemas com Bloqueador (ou Sustentador ou "Holder")



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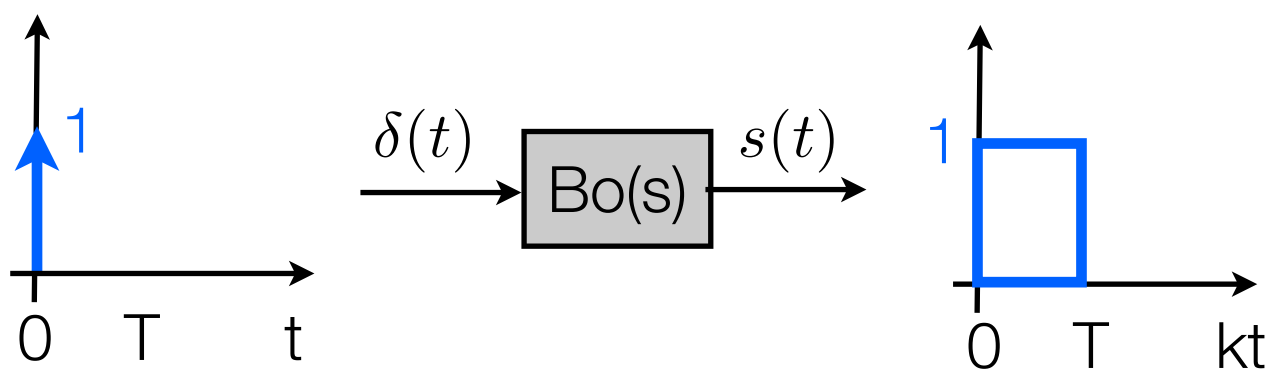
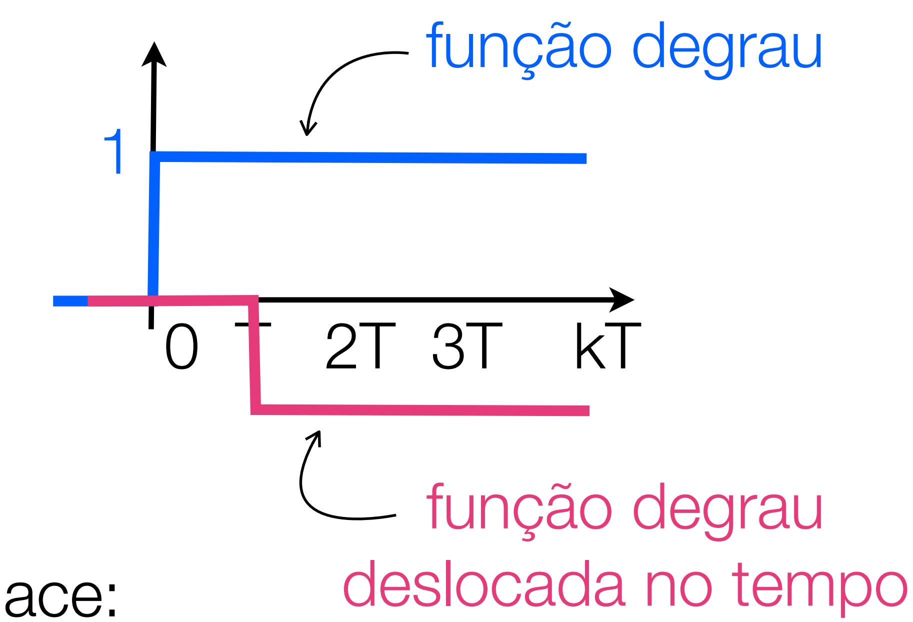




Lembrando das propriedades da Transformada de Laplace:

- Deslocamento no tempo: $\mathscr{L}\{f(t-a)\cdot u(t-a)\} = e^{-a\,s}\cdot F(s)$
- Função degrau: $\mathscr{L}\{u(t)\} = \dfrac{1}{s}$

Temos então:

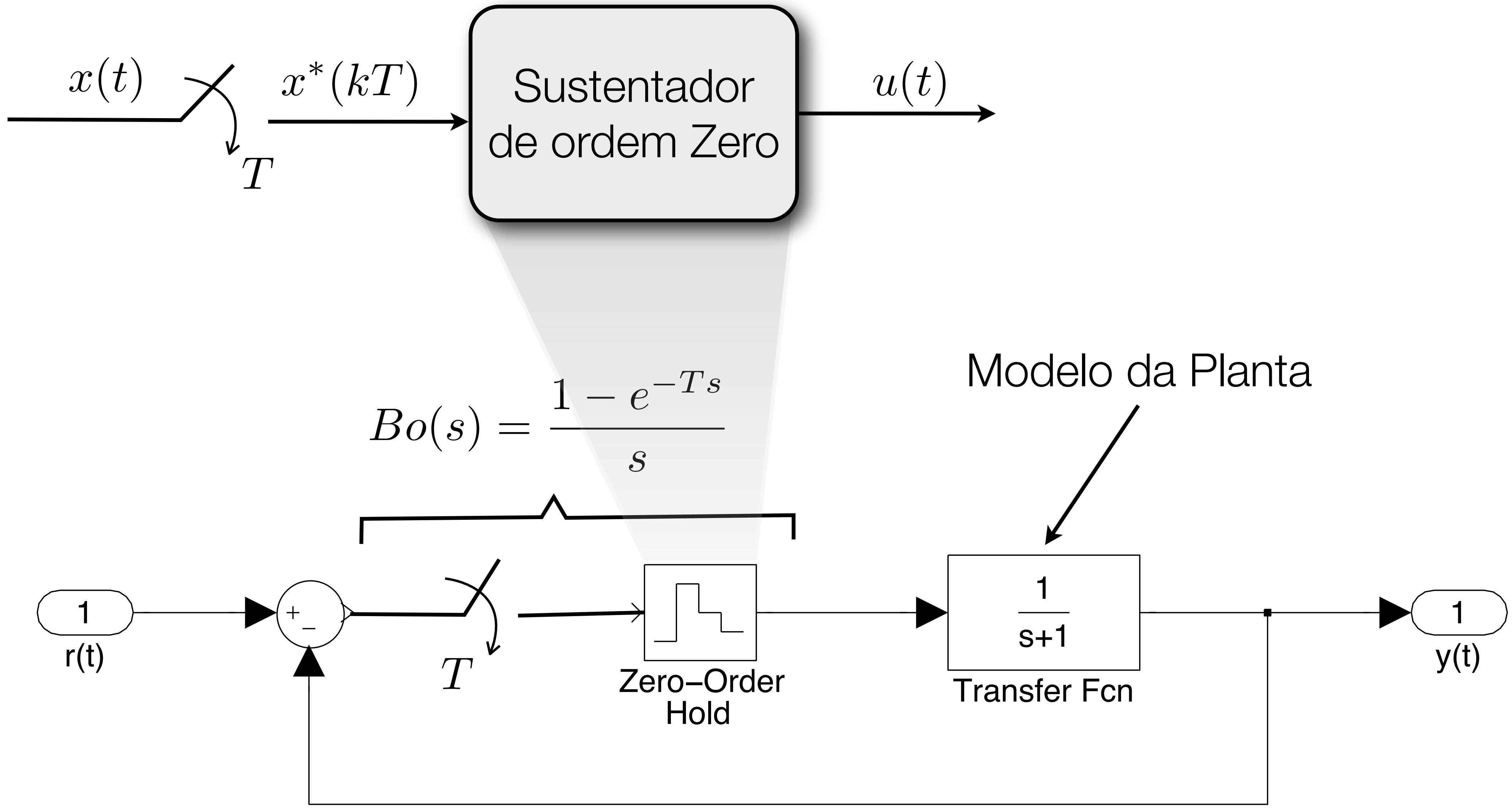
$$Bo(s) = \frac{S(s)}{E(s)}$$

$$e(t) = \delta(t) \quad \therefore \quad E(s) = 1$$

$$s(t) = u(t) - u(t-T)$$

$$Bo(s) = \frac{1}{s} - \frac{e^{-Ts}}{s}$$

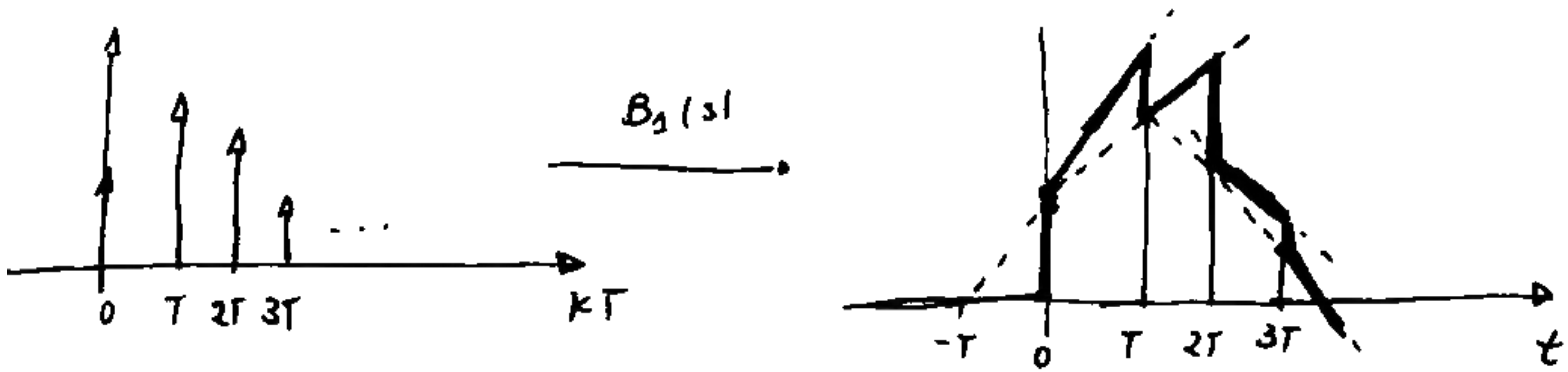
$$Bo(s) = \frac{1 - e^{-Ts}}{s}$$



Introdução do Sustentador em uma planta

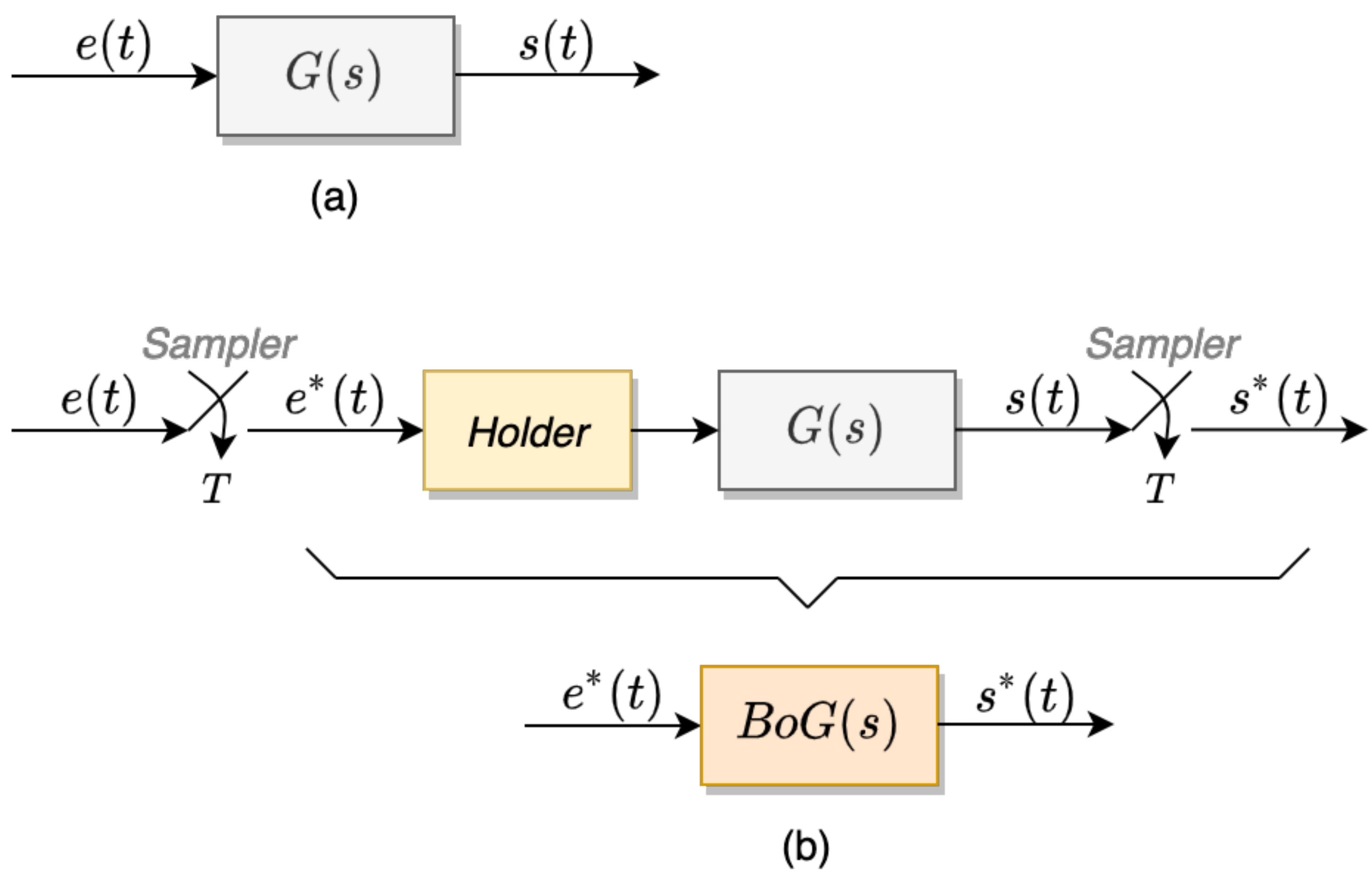
Modelo Final

Sustentador de 1ª-ordem:

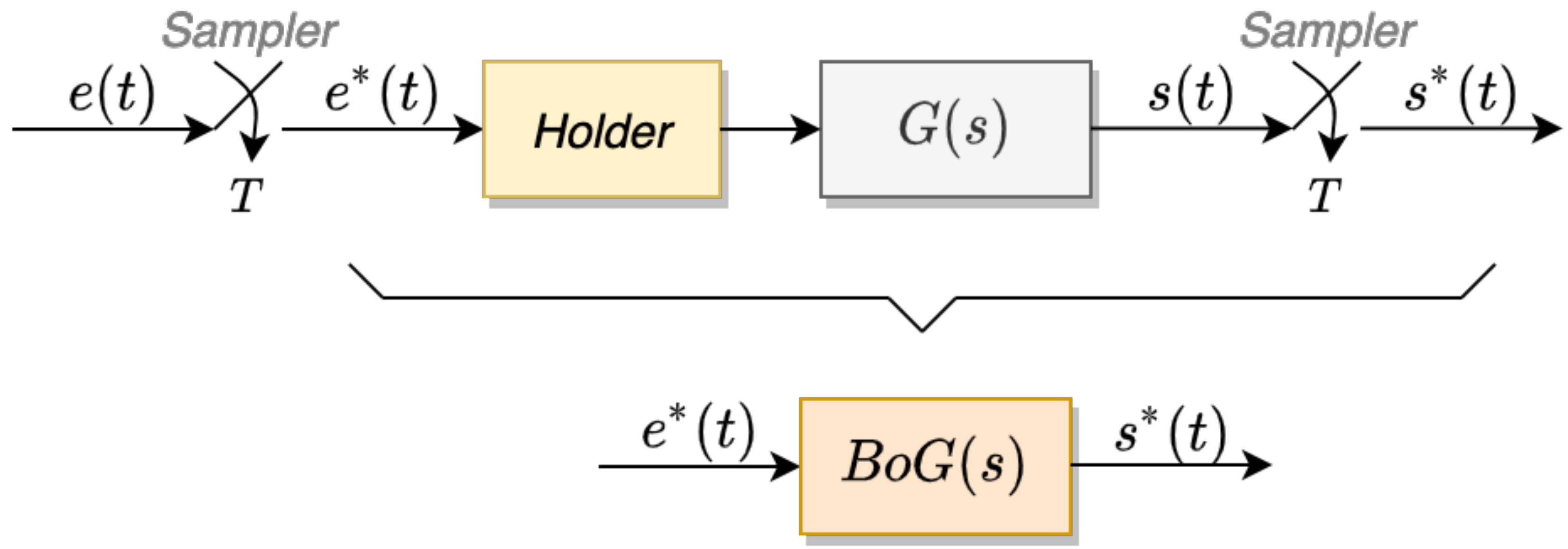


Outros Modelos de Sustentadores

Associando o Bloqueador a um processo...



- Como o sistema (a) pode ser visualizado depois de amostrado (b)



Note que:

$$\frac{S(s)}{E(s)} = Bo(s) \cdot G(s) \text{ e que: } \frac{S(z)}{E(z)} = Bo \cdot G(z) = \mathbb{Z}\left\{Bo(s) \cdot G(s)\right\}$$

Então: $\mathbb{Z}\left\{Bo(s) \cdot G(s)\right\} = \mathbb{Z}\left\{\left(\frac{1 - e^{-Ts}}{s}\right) \cdot G(s)\right\} = \mathbb{Z}\left\{\frac{G(s)}{s}\right\} - \mathbb{Z}\left\{\frac{e^{-Ts}\, G(s)}{s}\right\}$

Se $F(s) = \frac{G(s)}{s}$, temos: $\mathbb{Z}\left\{Bo(s) \cdot G(s)\right\} = \mathbb{Z}\left\{F(s)\right\} - \mathbb{Z}\left\{e^{-Ts}\, F(s)\right\}$

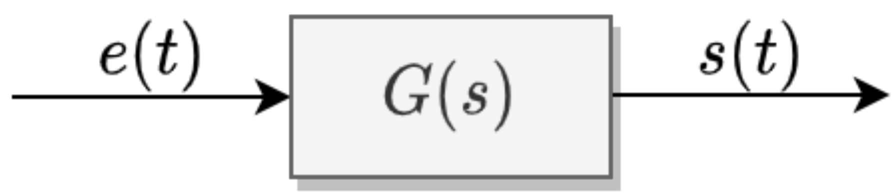
Como: $e^{-Ts} \cdot F(s)$ é equivalente a um atraso no tempo de um período de amostragem:

$e^{-Ts} \cdot F(s) \Rightarrow f(t - T)$, então: $\mathbb{Z}\left\{f[(k-1)T]\right\} = z^{-1} \cdot F(s)$

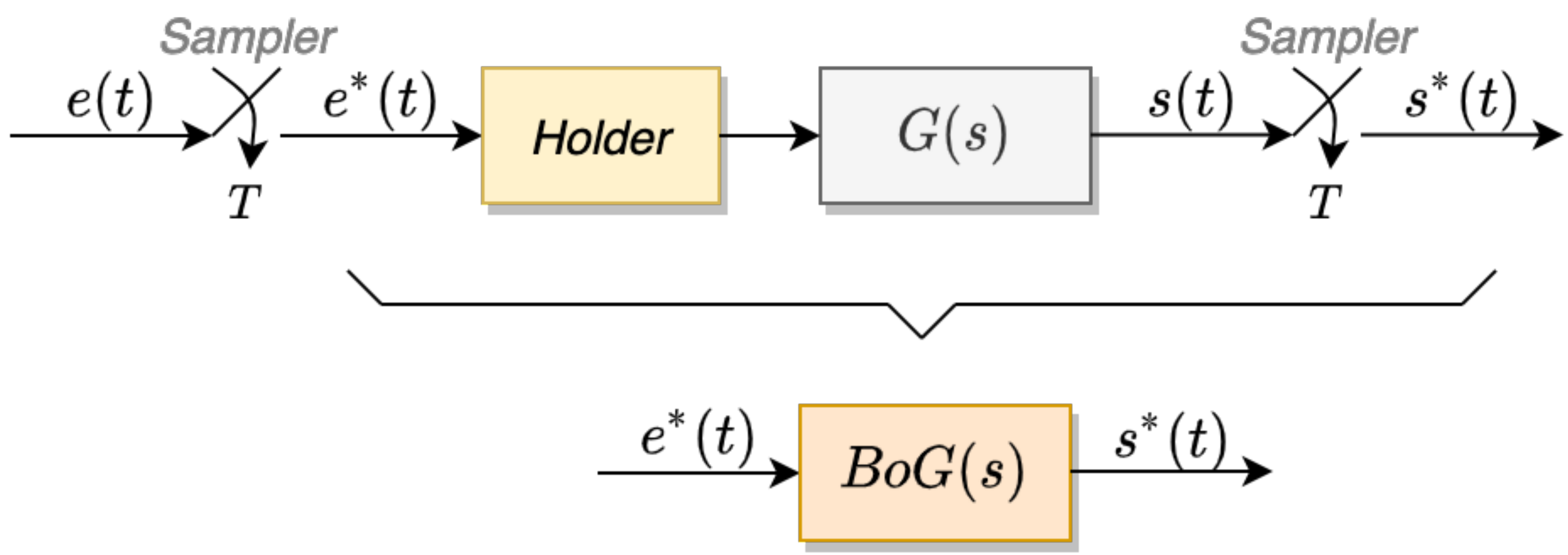
E assim: $\mathbb{Z}\left\{Bo(s) \cdot G(s)\right\} = \mathbb{Z}\left\{F(s)\right\} - z^{-1} \cdot \mathbb{Z}\left\{F(s)\right\}$ ou simplesmente:

$$BoG(z) = \left(1 - z^{-1}\right) \cdot \mathbb{Z}\left\{\frac{G(s)}{s}\right\}$$

$$\mathcal{Z}\left\{Bo(s)\cdot G(s)\right\} = \mathcal{Z}\left\{\frac{1-e^{-T\,s}}{s}\cdot G(s)\right\} = \mathcal{Z}\left\{\frac{G(s)}{s}\right\} - \mathcal{Z}\left\{\frac{e^{-T\,s}G(s)}{s}\right\}$$



(a)



(b)

$$BoG(z) = (1 - z^{-1})\cdot\mathcal{Z}\left\{\frac{G(s)}{s}\right\}$$

Exemplo_1) Obter BoG(z) para o sistema abaixo:

Se $G(s) = \dfrac{K \cdot g}{(1 + \tau s)}$ (um sistema de 1a-ordem).

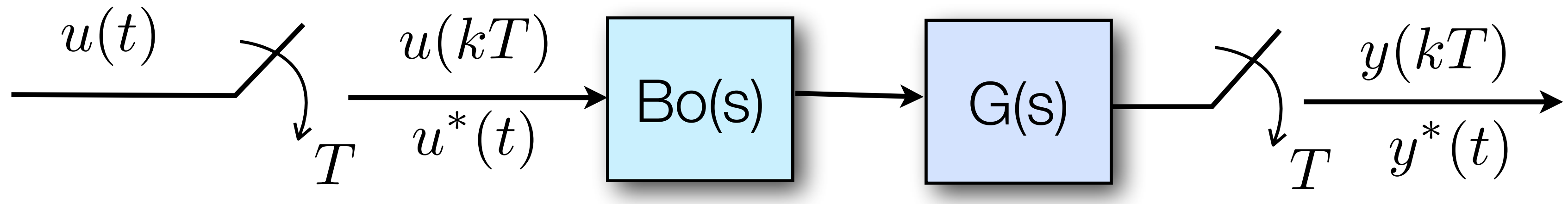
Aplicando: $BoG(z) = (1 - z^{-1}) \cdot \mathbb{Z}\left\{\dfrac{G(s)}{s}\right\}$

Obtemos: $\mathbb{Z}\left\{\dfrac{K(1 - e^{-Ts})\, g}{s(1 + \tau s)}\right\} = K\, g(1 - z^{-1}) \cdot \mathbb{Z}\left\{\dfrac{1}{s(1 + \tau s)}\right\}$

"Pausa" para Transformada Z...

Exemplo: BoG(z) de Sistema de 1a-Ordem...

- Seja o sistema:



Se $G(s) = \dfrac{K \cdot g}{(1+\tau s)}$ (Um sistema de 1a-ordem)

Aplicando o que acabamos de verificar: $BoG(z) = (1 - z^{-1}) \cdot \mathcal{Z}\left\{\dfrac{G(s)}{s}\right\}$, obtemos:

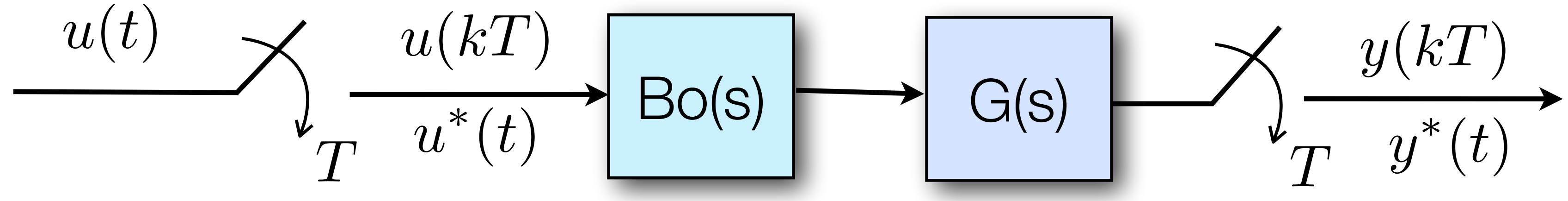
$$BoG(z) = \mathcal{Z}\left\{\frac{K \cdot g \cdot (1 - e^{-Ts})}{s(1+\tau s)}\right\} = (1 - z^{-1}) \cdot \mathcal{Z}\left\{\frac{K \cdot g}{s(1+\tau s)}\right\} = K\,g\,(1 - z^{-1}) \cdot \mathcal{Z}\left\{\frac{1}{s(1+\tau s}\right\}$$

$$\frac{1}{s(1+\tau s)} = \frac{A}{s} + \frac{B}{(1+\tau s)} = \frac{A(1+\tau s) + Bs}{s(1+\tau s)} = \frac{A + s(B + \tau A)}{s(1+\tau s}$$

$$A = \left.\frac{1}{(s+\tau s)}\right|_{s=0} = 1$$

$$B = \left.\frac{1}{s}\right|_{s={}^{1}/(-\tau)} = -\tau$$

$$\frac{1}{s(1+\tau s)} = \frac{1}{s} - \frac{\tau}{(1+\tau s)} = \frac{1}{s} - \frac{1}{{}^{1}/\tau + s}$$



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$$BoG(z) = \mathcal{Z}\left\{\frac{K \cdot g \cdot (1-e^{-Ts})}{s(1+\tau s)}\right\} = (1-z^{-1}) \cdot \mathcal{Z}\left\{\frac{K \cdot g}{s(1+\tau s)}\right\} = K\,g\,(1-z^{-1}) \cdot \mathcal{Z}\left\{\frac{1}{s(1+\tau s}\right\}$$

$$\frac{1}{s(1+\tau s)} = \frac{A}{s} + \frac{B}{(1+\tau s)} = \frac{A(1+\tau s)+Bs}{s(1+\tau s)} = \frac{A+s(B+\tau A)}{s(1+\tau s}$$

$$A = \left.\frac{1}{(s+\tau s)}\right|_{s=0} = 1$$

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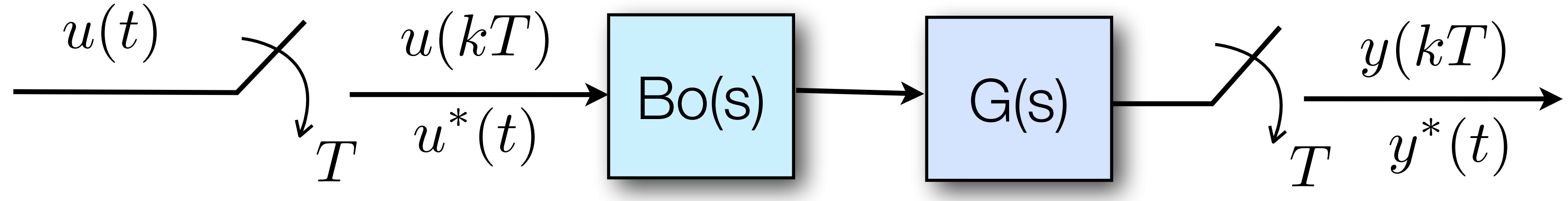
$$\frac{1}{s(1+\tau s)} = \frac{1}{s} - \frac{\tau}{(1+\tau s)} = \frac{1}{s} - \frac{1}{1/\tau + s}$$

Determinando a transformada Z:

$$\mathcal{Z}\left\{\frac{1}{s(s+\tau s)}\right\} = \frac{z}{z-1} - \frac{z}{z-e^{-T/\tau}} \quad \text{ou}$$

$$\mathcal{Z}\left\{\frac{1}{s(s+\tau s)}\right\} = \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1} \cdot e^{-T/\tau}}$$



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$$BoG(z) = \mathcal{Z}\left\{\frac{K \cdot g \cdot (1-e^{-Ts})}{s(1+\tau s)}\right\} = (1-z^{-1}) \cdot \mathcal{Z}\left\{\frac{K \cdot g}{s(1+\tau s)}\right\} = K\, g\,(1-z^{-1}) \cdot \mathcal{Z}\left\{\frac{1}{s(1+\tau s}\right\}$$

$$\mathcal{Z}\left\{\frac{1}{s(s+\tau s)}\right\} = \frac{z}{z-1} - \frac{z}{z-e^{-T/\tau}} \quad \text{ou}$$

$$\mathcal{Z}\left\{\frac{1}{s(s+\tau s)}\right\} = \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1} \cdot e^{-T/\tau}}$$

$$BoG(z) = \frac{K\, g\,(1-z^{-1}) \cdot 1}{(1-z^{-1})} - \frac{K\, g\,(1-z^{-1}) \cdot 1}{(1-z^{-1} \cdot e^{-T/\tau})}$$

$$BoG(z) = Kg - \frac{Kg(z-1)}{(z-e^{-T/\tau})} = \frac{Kg(z-e^{-T/\tau}) - Kg(z-1)}{(z-e^{-T/\tau})} = \frac{Kg\,z - Kg\,e^{-T/\tau} - Kg\,z + Kg}{(z-e^{-T/\tau})}$$

$$BoG(z) = \frac{K\, g\,(1-e^{-T/\tau})}{(z-e^{-T/\tau})}$$

Note: 1ª-ordem com integrador

Problema

- Determine BoG(z) para: $G(s) = \dfrac{K}{s(s+a)}$

$$BoG(z) = \mathcal{Z}\left\{\frac{K(1-e^{-Ts})}{s^2(s+a)}\right\} = K(1-z^{-1})\cdot\mathcal{Z}\left\{\frac{1}{s^2(s+a)}\right\}$$

- Resposta:

$$BoG(z) = \frac{K\left[z(aT+e^{-aT}-1)+(1-aTe^{-aT}-e^{-aT})\right]}{a^2(z-1)(z-e^{-aT})}$$