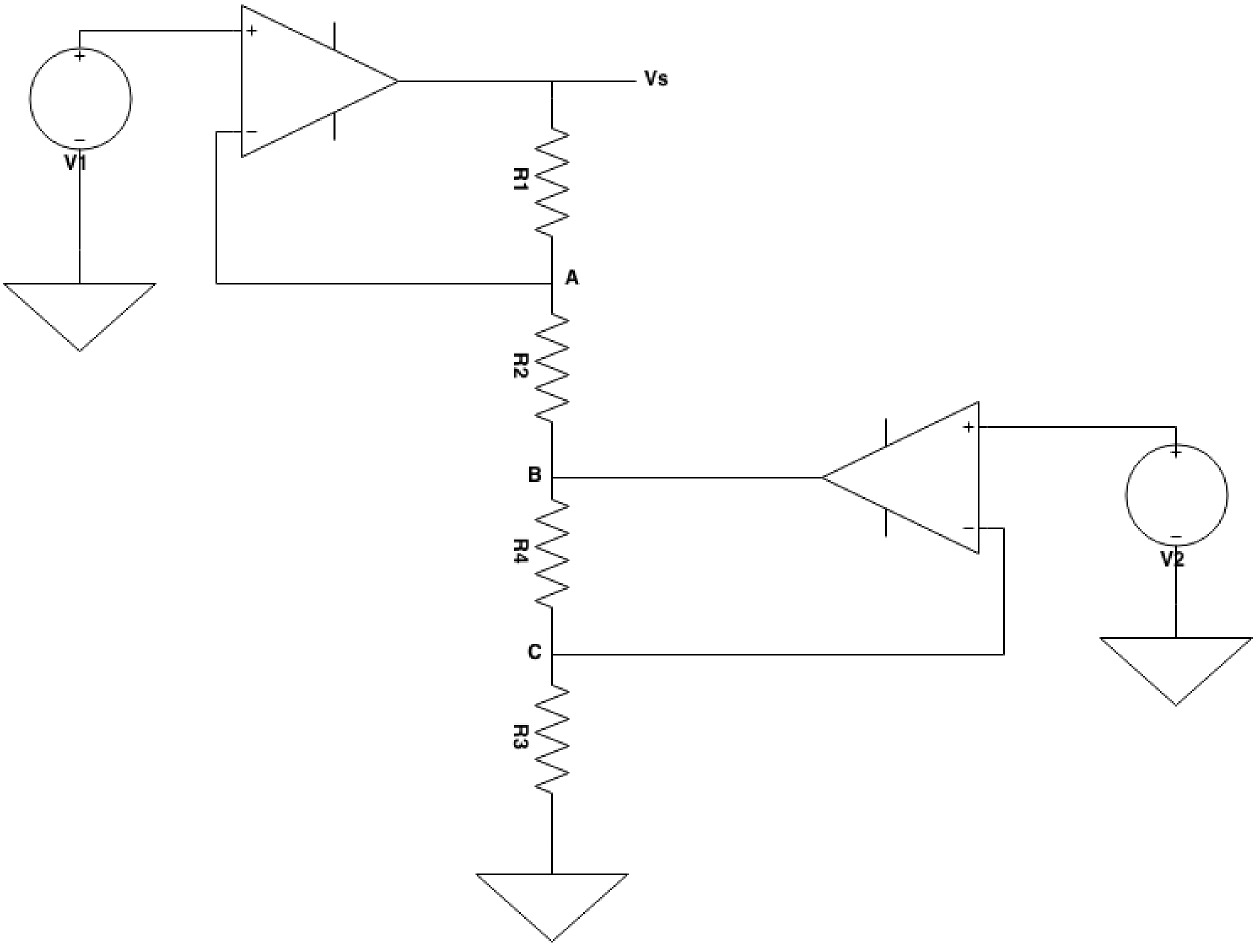
ELECTRICALENGINEERING

Transfer function of an op amp circuit

Asked 7 years, 9 months ago Modified 7 years, 9 months ago Viewed 2k times

2

Let's suppose the following circuit:



I am trying to find a relation between Vs, V1 and V2 at this circuit. Please note that Vs is measured in relation to the ground. I have declared some extra points (A,B,C) on the circuit and using voltage divider laws and the properties of an ideal operational amplifier I came up with the following solution:

$$V_2 = V_C = V_B \frac{R_3}{R_3+R_4} \Rightarrow V_B = \frac{V_2}{\frac{R_3}{R_3+R_4}} \quad (1)$$

$$V_1 - V_B = (V_S - V_B)\frac{R_2}{R_1+R_2} \quad (2)$$

$$(2) \overset{(1)}{\Rightarrow} V_1 - \frac{V_2}{\frac{R_3}{R_3+R_4}} = \left(V_S - \frac{V_2}{\frac{R_3}{R_3+R_4}}\right)\frac{R_2}{R_1+R_2}$$

$$\Rightarrow V_1 - \frac{V_2\cdot(R_3+R_4)}{R_2} = V_S \cdot \frac{R_2}{R_1+R_2} - \frac{V_2(R_3+R_4)}{R_3}\cdot\frac{R_2}{R_1+R_2}$$

$$\Rightarrow V_S = \frac{\frac{V_2(R_3+R_4)}{R_2} - V_1 - \frac{V_2(R_3+R_4)}{R_3}\cdot\frac{R_2}{R_1+R_2}}{\frac{R_2}{R_1+R_2}}$$

Is this correct?

Another solution with a different result is the following:

$$V_C = V_2 \quad (1)$$

$$i_1 = \frac{V_C}{R_3} \overset{(1)}{=} \frac{V_2}{R_3} \quad (2)$$

$$i_2 = i_1 \overset{(2)}{=} \frac{V_2}{R_3} \quad (3)$$

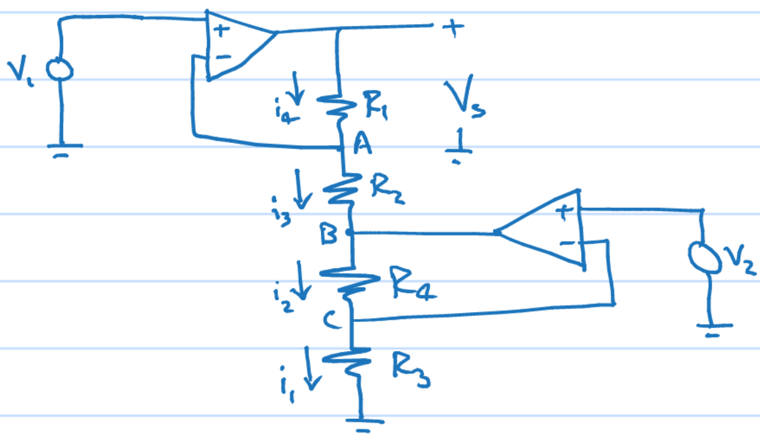
$$V_A = V_1 \quad (4)$$

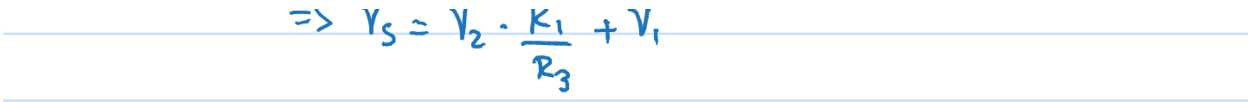
$$i_3 = i_2 \overset{(3)}{=} \frac{V_2}{R_3} \quad (5)$$

$$i_4 = i_3 \quad (6)$$

$$i_4 = \frac{V_S - V_A}{R_1} \underset{(4)}{\overset{(5),(6)}{\Longrightarrow}} \frac{V_2}{R_3} = \frac{V_S - V_1}{R_1}$$

$$\Rightarrow V_2 \cdot \frac{R_1}{R_3} = V_S - V_1$$





I am really confused.

operational-amplifier | ground | voltage-divider | transfer-function

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edited Dec 31, 2014 at 15:13

asked Dec 30, 2014 at 21:19

mgus
227 2 6 14

The second derivation is wrong: you write `i3=i2` at node B, but you neglect the current from the output of the lower op-amp, which is not negligible in general. – Lorenzo Donati support Ukraine Dec 31, 2014 at 21:31

2 Answers

Sorted by: Highest score (default)

3

Voltage division isn't a great approach to hang this on. Just build the output from the bottom up. You just need to know that current doesn't enter the input terminals of an op amp.

You know Vc, so you know the current through R3. That has to be the same as the current through R4, so now you know Vb. You also know Va, so now you can calculate current through R2, which has to be the same as the current through R1, which would give you Vs.

Looks a little like the input stage of an instrumentation amplifier,but there's an extra resistor. **CORRECTION** This is a two op-amp instrumentation amplifier. Full discussion of the circuit at http://www.analog.com/static/imported-files/design_handbooks/5812756674312778737Complete_In_Amp.pdf on page 2-4.

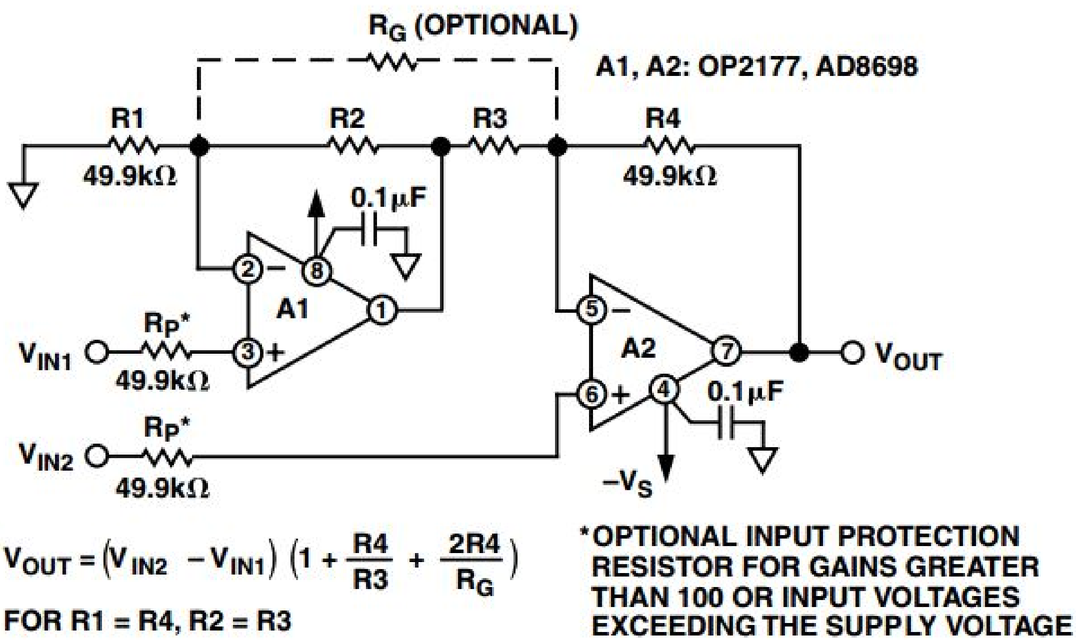


Figure 2-6. A 2-op amp in-amp circuit.

-- though the resistor numbers and inputs are not quite the same as what you use

Taking a shot at your derivation, starting from the line i2=i1=V2/R3, lets have a go at it

$$V_B = V_2 + i_2 R_4 = V_2 + \frac{V_2 R_4}{R_3}$$

$$= V_2 \left(1 + \frac{R_4}{R_3}\right)$$

$$i_3 = \frac{V_A - V_B}{R_2} = \frac{V_1 - V_2 \left(1 + \frac{R_4}{R_3}\right)}{R_2}$$

$$V_S = V_1 + i_4 R_1 = V_1 + \frac{V_1 R_1}{R_2} - V_2 \frac{R_1}{R_2} \left(1 + \frac{R_4}{R_3}\right)$$

or just shifting to make this look a bit more differential:

$$V_S = V_1 \left(1 + \frac{R_1}{R_2}\right) - V_2 \frac{R_1}{R_2} \left(1 + \frac{R_4}{R_3}\right)$$

That's a quick pass, and something feels wrong. Feel free to correct.

Share Cite Follow edited Dec 31, 2014 at 18:47 answered Dec 30, 2014 at 22:32

Scott Seidman
28.3k 4 42 105

Thanks @acd -- I'm having a terrible time w/ a new autocorrect. – Scott Seidman Dec 30, 2014 at 22:50

I followed your advice and posted what I found (see above). But I ended up with a completely different result. Can you check it please? – mgus Dec 31, 2014 at 15:17

@KonstantinosKonstantinidis Will do. Editing Now – Scott Seidman Dec 31, 2014 at 18:20

Whew, TeX workout for someone who doesn't know it! – Scott Seidman Dec 31, 2014 at 18:39

I didn't notice anything wrong about your solution. But what I still don't understand is why voltage division I initially used gets us to a different solution. What is wrong about this? – mgus Dec 31, 2014 at 23:19

3

A clever circuit... beautiful and symmetrical (usually, R1 = R2 = R3 = R4 = R) like all the circuits of instrumentation amplifiers... It also gives a good opportunity to show how to make the unfamiliar circuits familiar - by dividing them into more well-known functional blocks instead of blindly analyzing them...

Structure. We can discern in this circuit of a *perfect instrumentation amplifier* two sub-circuits - an *imperfect (unbalanced) differential amplifier* (the top part consisting of the upper op-amp and the resistors R1, R2), and an ordinary *non-inverting amplifier* (the bottom part consisting of the lower op-amp and the resistors R3, R4).

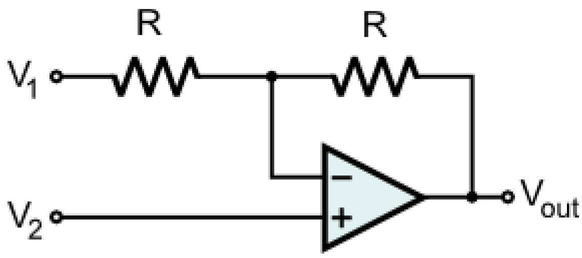
Analysis. Let's first consider the upper circuit part. With respect to V1, it is a non-inverting amplifier with gain of 2, and with regard to the lower input (from the side of the lower non-

inverting amplifier) - an inverting amplifier with gain of -1. As the lower non-inverting amplifier has a gain of 2, the two gains (inverting and non-inverting) of the imperfect differential amplifier are equalized... the two partial voltages superimpose and mutually neutralize at the op-amp output... and it becomes a perfect balanced differential (instrumentation) amplifier. From this perspective, the analysis is very simple:

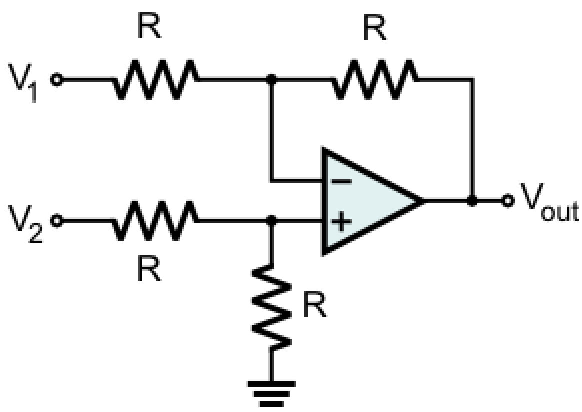
Vs = V1*(R1 + R2)/R2 - V2*(R3 + R4)/R3*R1/R2 = 2V1 - 2V2 = 2(V1 - V2); Happy New Year!

Philosophy. It is interesting to reveal the evolution of the op-amp differential amplifier to see where this circuit solution stays. I will use figurative (not generally accepted) names of the particular circuit solutions that are more meaningful. Also, to simplify this qualitative explanation, I suppose equal resistances (R).

1. ***Unbalanced "differential amplifier"***. To make a differential amplifier, we need simply to subtract two input voltages. First we introduce a negative feedback by two resistors to obtain a fixed stable gain and then apply the two voltages to the inverting and non-inverting inputs of this "differential amplifier". Here the low resistance of the inverting input is a problem... but the big problem is that the two gains are not equal - the inverting gain (1) is less than the non-inverting gain (2). So, we have two choices to equalize them - to decrease (two times) the non-inverting gain or to increase (two times) the inverting gain. Let's consider them below...



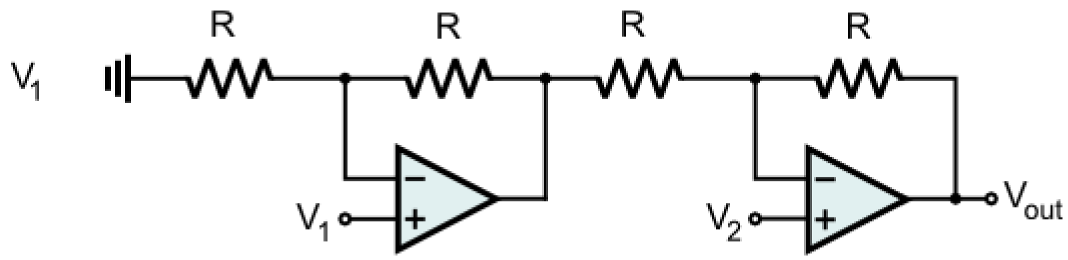
2. ***Differential amplifier with non-inverting attenuation***. To decrease (two times) the non-inverting gain, we can connect a voltage divider (with two equal resistors) before the non-inverting input thus obtaining the classical 1-op-amp differential amplifier. The two gains are now equalized... but the high resistance of the non-inverting input is decreased...



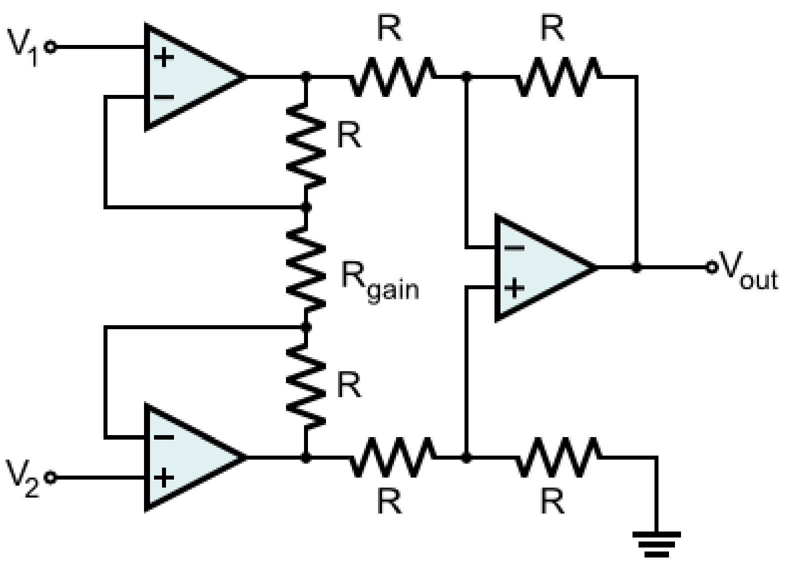
3. ***Differential amplifier with inverting amplification***. With the same success we can

increase (two times) the inverting gain if we connect a non-inverting amplifier (with a gain of 2) before the inverting input (the discused here solution). The two gains are equalized

again... and the both inputs have high resistance... It is a real 2-op-amp instrumentation amplifier.



4. ***Buffered differential amplifier with non-inverting attenuation***. Finally, we can modify the classic 1-op-amp differential amplifier (case 2) by including non-inverting amplifiers before its inputs; this will solve the problems of the low input resistances. If we are inventive enough, we will combain the two lower resistors of the voltage dividers (inside the input non-inverting amplifiers) into one resistor (Rgain) that can regulate simultaneosly both the input gains. Thus we will obtain the classic 3-op-amp instrumentation amplifier. It is interesting that there is a virtual ground in the middle point of Rgain; it has replaced the real ground.



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edited Jun 11, 2020 at 15:10
Community Bot
1

answered Dec 31, 2014 at 10:28
Circuit fantasist
9,538 1 16 37

This doesn't seem right, given the circuit from the AD handbook I put up. – Scott Seidman Dec 31, 2014 at 18:44

Well, I will consider it... but now I have to go back to the New Year's table... the New Year comes here after three hours... Cheers! – Circuit fantasist Dec 31, 2014 at 19:34

Happy New Year!! – Scott Seidman Dec 31, 2014 at 20:13

@Scott Seidman, I have decided to start the new year by a fancy story about the evolution of the op-amp differential amplifier. BTW, as drawn, the AD circuit diagram is more appropriate for the intuitive understanding... – Circuit fantasist Jan 1, 2015 at 15:00

1 @Circuitfantasist - See my reworking of your (so far) deleted without trace answer at address below. Seemed too good to let it go unremarked off into the void. electronics.stackexchange.com/questions/146442/... – Russell McMahon ♦ Jan 4, 2015 at 7:35